Optimal Tariffs in a World Shaped by Multinational Production*

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Abstract

Multinational firms account for roughly 60 percent of global exports, yet their implications for optimal tariffs remain unclear. We study unilateral import tariffs in a multisector trade model with multinational production and input–output linkages. Multinational production pulls the optimal tariff in two directions. The option to relocate production makes trade more responsive to tariffs, which limits how far a tariff can push down the prices a country pays for its imports. But firms that concentrate production in one country despite having alternatives reveal strong advantages there, so trade responds less than a standard trade model predicts. In our calibration, the optimal uniform U.S. tariff is 27 percent — 21 percentage points below what it would be if firms could not produce abroad, but 5 points above what a standard trade model recommends. A uniform tariff captures more than 99 percent of the welfare gain from tariffs that vary by country and sector.

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Introduction

Multinational enterprises account for approximately 60 percent of global exports (OECD 2026). However, most analyses of optimal trade policy abstract from firms having the option and ability to use the same technology to produce the same product in different countries. The U.S. experience with washing-machine tariffs illustrates why this omission matters for trade policy. In 2012, the United States imposed antidumping and countervailing duties on washing machines from Korea and Mexico, aimed at Korean manufacturers such as Samsung and LG. In response, the firms moved production to China. When duties were extended to Chinese-made machines in 2016, they moved it again, to Thailand and Vietnam. Consumer costs fell or changed little during this time (Flaaen, Hortaçsu, and Tintelnot 2020). In 2018, the United States extended tariffs to washing machines from nearly all foreign origins. With few untariffed foreign washing machines left, consumer costs rose, despite Samsung and LG having begun producing in the United States following investment announcements in 2017 (Flaaen, Hortaçsu, and Tintelnot 2020). The same firms thus responded to each round of tariffs by changing where they produced. Consequently, what a tariff achieved depended on which production locations it covered and the subsequent relocation of production. Trade models in which technology is tied to the country of production cannot capture this response, and so cannot tell us how the ability to relocate production affects the optimal tariff. Hence, this paper asks: how does the ability of firms to choose where to produce change the level and structure of optimal import tariffs?

I study optimal import tariffs in a Ricardian model of trade and multinational production with multiple countries, sectors, and input–output linkages. A country's technology can be used to produce at home or abroad, so the model distinguishes the technology-owning country, the producing country, and the buying country. The importing government chooses tariffs by production origin and sector, irrespective of technology ownership, to maximize household real consumption. Other countries hold their policies fixed and do not retaliate, so the policy problem isolates the terms-of-trade motive for import tariffs.

Multinational production affects the optimal tariff in two opposing ways. First, allowing production abroad adds a margin of adjustment that weakens the terms-of-trade motive. The optimal uniform U.S. tariff falls from 48.1% under prohibitive multinational production costs to 27.2% under calibrated costs. Second, accounting for observed multinational production raises the optimal tariff from 22.0% in the trade-only benchmark to 27.2% in the MP model. Both comparisons start from the same baseline bilateral allocation, so the differences arise from how that allocation responds to trade policy.

These findings answer two distinct questions: what changes when firms gain the option to produce abroad, and what does a trade model miss when it ignores how firms already use that option? Using a two-country, one-sector special case, I show that both answers operate through a single object, the foreign export-supply elasticity. A tariff reduces demand for foreign production and puts downward pressure on foreign wages, which lowers the pre-tariff price the importing country pays for the goods it continues to import. How large this terms-of-trade gain is depends on the export-supply elasticity. The foreign export-supply elasticity measures how much the tariffed country's production for export responds to changes in its production costs relative to those in the importing country. When export supply is less responsive, more of the adjustment to a tariff occurs through prices rather than quantities, and the importing country obtains a larger price reduction for a smaller reduction in imports. In this special case, an interior optimal tariff equals the inverse of this elasticity at the optimum. Multinational production does not change this terms-of-trade motive; it changes the elasticity, and hence the optimal tariff, by changing how readily trade adjusts to

relative costs. The two questions then correspond to two distinct forces. Making production in alternative countries feasible gives firms an additional margin of adjustment, relocating production, through which trade can respond to a tariff without a change in relative prices. Observing how firms use the locations available to them has the opposite implication. When firms concentrate production in one country despite available alternatives, this reveals a strong production advantage there, and a larger change in relative costs is needed to induce relocation.

I formalize these forces by studying differences in the export-supply elasticity in the two-country, one-sector special case in which the bilateral expenditure shares coincide at the baseline equilibrium, so that differences in the elasticity reflect differences in adjustment margins rather than in the underlying allocation. The first counterfactual lowers multinational-production costs from prohibitive to finite levels while holding the dispersion of marginal productivity fixed. A technology whose home production becomes relatively more costly can now serve buyers from another location. This additional margin weakly raises the export-supply elasticity and weakens the terms-of-trade incentive for protection. The second counterfactual asks what ownership-production data reveal beyond bilateral trade flows. The same bilateral import share can arise when each technology source uses a similar mix of production locations, or when individual sources concentrate in different countries. Sources that concentrate production in different locations reveal different net advantages, reflecting productivity and the costs of producing abroad and reaching buyers, and bilateral trade shares alone conceal these differences. Holding the cost sensitivity of location choices within a technology source equal to the trade-only model's trade elasticity, I show that accounting for these choices weakly lowers the export-supply elasticity and strengthens the terms-of-trade incentive. More generally, I derive a condition showing how the comparison depends jointly on productivity parameters and source-specific production choices.

These analytical results compare tariff incentives at a common allocation, but optimal tariff levels must be solved jointly with the equilibrium they induce: as tariffs change, so do the expenditure shares and technology-source composition that determine the export-supply elasticity. I therefore quantify both counterfactuals in the multi-country model, which combines the multinational-production framework of Ramondo and Rodríguez-Clare (2013) with multiple sectors and input–output linkages, following Caliendo and Parro (2015) and Alviarez (2019). Productivity is correlated across potential production locations, and under perfect competition each good is supplied through the lowest-cost combination of technology source and production location. Buyers can therefore be supplied by exports from the technology's source country, by local affiliate production, or by an export platform in a third country. Production combines labor and intermediate inputs, with labor mobile across sectors but immobile across countries, so a tariff on an imported intermediate changes downstream production costs as well as the purchaser's sourcing decision. A tariff on one foreign supplier can also change that supplier's wage relative to the wages of competing exporters, shifting expenditure between them. These responses create scope for tariffs to vary across origins and sectors. Households receive labor income and rebated tariff revenue, and welfare is real consumption. The government weighs the terms-of-trade gain against distortions to sourcing and production, taking into account the full adjustment of wages, prices, and expenditure throughout the network.

I calibrate the model using data on production ownership and Inter-Country Input-Output tables for 2020 from the OECD Analytical Activities of Multinational Enterprises database, aggregated to 45 countries and 33 sectors (Cadestin et al. 2018). The input–output accounts determine bilateral sales, labor and intermediate-input cost shares, and final-demand shares. I combine these accounts with output by ownership and production country to recover trilateral flows consistent with both margins, and use these flows to

construct a model-consistent, balanced-trade baseline at zero tariffs. Following Dekle, Eaton, and Kortum (2008) and Caliendo and Parro (2015), I use exact-hat algebra to express tariff-induced changes in equilibrium as functions of baseline allocations and productivity parameters. Both counterfactual economies therefore reproduce the same bilateral expenditure shares and output with common input-cost and final-demand shares. The comparison thus holds starting allocations fixed while allowing adjustment mechanisms to differ. In the first counterfactual, the difference is whether a technology can be used in another country. In the second, it is whether ownership-production information disciplines the response inferred from the same bilateral trade flows.

I fix the correlation of productivity across potential production locations at 0.55, following Arkolakis et al. (2018). The within-sector distribution of productivity is then targeted using the empirical sectoral trade elasticities from Caliendo and Parro (2015). The mapping matches the shared cost sensitivity of location choices across technology-source countries. This differs from the model-implied sectoral trade elasticity, which retains the full endogenous sourcing response associated with changes in technology-source composition across counterfactual equilibria and tariff calculations.

The exporter–sector results preserve the distinction between the two counterfactuals. Allowing multinational production lowers tariffs for most origin–sector pairs while changing their relative pattern little; the expenditure-weighted Spearman rank correlation between the two tariff vectors is 0.946. Accounting for observed multinational production instead primarily raises the level of protection; the corresponding weighted rank correlation with the trade-only tariff schedule is 0.943. Within each model, tariffs vary substantially across individual origin–sector pairs. Yet this dispersion adds little welfare relative to a uniform tariff. In the multinational-production model, the optimal uniform tariff raises U.S. welfare by 0.497% relative to free trade, compared with 0.500% under the fully differentiated schedule, an additional gain of 0.003 percentage points. Multinational production therefore substantially changes the optimal level of protection, while a uniform tariff captures almost all the welfare gains available from differentiation. This holds even though the differentiated schedule can exploit both sectoral production-network effects and exporter-specific wage responses.

This paper contributes to the literature on optimal trade policy with multinational production. For a broad review of optimal trade policy, see Caliendo and Parro (2022). Wang (2017) studies firm-specific trade and FDI policies in a two-country model based on Helpman, Melitz, and Yeaple (2004) in which heterogeneous firms choose between exporting and producing through foreign affiliates. Gutiérrez et al. (2025) study how endogenous selection into multinational production, market power, and transfer pricing shape trade responses and optimal import tariffs. Their double-CES, labor-only model distinguishes local firms from global firms that operate in every production location, abstracting from heterogeneity in multinationals' plant-location choices. Their analytical tariff results use a small-open-economy limit, and their quantitative results use a uniform tariff. Li, Li, and Lu (2026) characterize the optimal trade policy that jointly targets both the technology source and production location of goods in the Ramondo and Rodríguez-Clare (2013) model of trade and multinational production. Their instruments allow the government to exploit its position as a buyer of foreign production, a seller of domestic output, and a source of technology used abroad. I hold the policy experiment fixed across models: the government chooses unilateral import tariffs by production location while foreign policies remain unchanged. I also calibrate the economies to the same baseline bilateral trade flows, output, and input–output structure. Holding the baseline bilateral allocation and policy instrument fixed allows me to distinguish the adjustment made possible by production abroad from the adjustment inferred from observed multinational production. Allowing tariffs to vary by exporter

and sector then shows how these mechanisms affect both the level and the structure of protection.

The comparison also connects the paper to work on the policy relevance of microeconomic heterogeneity. Costinot et al. (2015) and Costinot, Rodríguez-Clare, and Werning (2020) show that models with similar aggregate gravity implications can prescribe different good- or firm-specific policies when taxes can target heterogeneity concealed by aggregation. In the Ricardian setting of Costinot et al. (2015), comparative advantage shapes differentiated export wedges, while the import-tax component is uniform. In the monopolistic-competition setting of Costinot, Rodríguez-Clare, and Werning (2020), selection instead creates a motive for lower import taxes on marginal foreign exporters. Related work studies how labor-market structure and international technology diffusion change optimal trade policy (Bai, Lu, and Wang 2024; Bai et al. 2025). Here, the multinational production underlying aggregate bilateral trade can change optimal tariffs even though the policy does not target multinationals. Ownership-production data reveal technology-source location advantages that bilateral trade shares conceal. Accounting for these advantages changes the export-supply response and optimal tariffs, while import duties remain the same for all firms producing in a given origin and sector.

The quantitative analysis contributes to work on how tariffs should vary across sectors in production networks. In a two-country tariff-only problem, Beshkar and Lashkaripour (2020) show that input–output linkages create a motive for sectoral differentiation because a tariff on an imported intermediate can propagate into downstream production and export prices. Such tariffs can partially substitute for unavailable export taxes by shifting some of their burden to foreign consumers, while also distorting domestic input choices. Antràs et al. (2024) provide a distinct efficiency rationale for sector targeting: with increasing returns in final-good production, tariff escalation—higher tariffs on final goods than on imported inputs—can raise welfare by expanding downstream activity. Their U.S.–China application attributes the welfare gains from the observed tariff changes overwhelmingly to final-good tariffs. Related quantitative work examines how markup distortions shape optimal tariffs (Lashkaripour 2021) and how scale economies and markups interact with trade and industrial-policy instruments (Lashkaripour and Lugovskyy 2023). I incorporate multinational production into a constant-returns quantitative trade model with input–output linkages, allowing source-specific location advantages to change how sectoral tariffs affect sourcing, production costs, and wages throughout the network.

A central question is how much additional welfare can be gained by differentiating tariffs across foreign suppliers within a sector. Prior quantitative analyses often simplify partner-level relative wage adjustments: the tariff-only quantitative exercise in Beshkar and Lashkaripour (2020) aggregates foreign countries into a single foreign entity, the main sufficient-statistics formulas in Lashkaripour (2021) approximate away welfare effects operating through relative foreign wage changes, and Lashkaripour and Lugovskyy (2023) restrict relative wages within the rest of the world. In contrast, I retain the complete vector of general-equilibrium wage responses, allowing tariffs on one supplier to reallocate production and expenditure through both trade and input–output linkages. Computationally, this complements Wang et al. (2025), who use automatic implicit differentiation for high-dimensional trade policy problems. By exploiting the structure of the equilibrium in Ricardian models and combining analytical partial derivatives with an adjoint method, I compute the exact general-equilibrium welfare gradient in minutes. The quantitative results reveal that, even when policy can fully exploit origin-specific wage movements and production-network linkages, tariff differentiation across origin–sector pairs yields negligible welfare gains beyond a uniform tariff.

The remainder of the paper is organized as follows. Section 1 presents the multicountry, multisector model.

Section 2 derives the two-country analytical results. Section 3 states the quantitative tariff problem and describes the computational method and the scope for country–sector differentiation. Section 4 describes the data and calibration. Section 5 presents the quantitative results. Section 6 conducts robustness exercises.

1 Model

The framework combines the multinational-production structure of Ramondo and Rodríguez-Clare (2013) with the multisector production network in Caliendo and Parro (2015). The economy consists of N countries and S sectors. Labor is the only factor of production and it is mobile across sectors and immobile across countries. L_n is the inelastically supplied labor in country n. Details on the derivation of the price indices and expenditure shares are in Appendix A.

1.1 Technology

Each sector s is composed of a continuum of differentiated goods or varieties $\omega \in [0,1]$ that do not overlap across sectors. Multinational production allows the source of the technology used to produce the good to differ from the production location. A good can be produced in country l using technology from country i and delivered to country n. One can think of a firm headquartered in i as using its technology to produce in l to supply demand in n.

Using technology from i in l entails an iceberg multinational-production cost $\gamma_{il,s} \geq 1$, with $\gamma_{ii,s} = 1$. Shipping the resulting good from l to n entails an iceberg trade cost $\tau_{ln,s} \geq 1$, with $\tau_{ll,s} = 1$. Country n levies an ad valorem tariff $t_{ln,s} \geq 0$ on goods produced in $l \neq n$, irrespective of the technology source. A destination can therefore be served by production in the technology-source country ($l = i$), local multinational production ($l = n \neq i$), or an export platform in a third country ($l \notin \{i, n\}$).

Fix a sector s and destination n. Let $u_{l,s}$ denote the cost of the labor and intermediate inputs to produce in country l and $a_{il,s}(\omega)$ the productivity of technology i when used to produce good ω in l. The tariff-inclusive marginal cost is

$$c_{iln,s}(\omega) = \gamma_{il,s}\tau_{ln,s}(1 + t_{ln,s})\frac{u_{l,s}}{a_{il,s}(\omega)}.$$

Multinational production abroad can lower this cost if higher productivity or cheaper inputs offset the multinational-production cost. Similarly, lower shipping costs and tariffs when supplying n can also raise the benefits of multinational production. Under perfect competition, the price paid in n is the lowest delivered cost across technology sources and production locations:

$$p_{n,s}(\omega) = \min_{i,l}\left\{\frac{\gamma_{il,s}u_{l,s}}{a_{il,s}(\omega)}\tau_{ln,s}(1 + t_{ln,s})\right\}. \tag{1}$$

Following Ramondo and Rodríguez-Clare (2013), each technology source i draws its productivity for producing sector s good ω across production locations l from a multivariate Fréchet distribution. The vectors of productivity draws, given by $\boldsymbol{A}_{i,s}(\omega) = \{A_{il,s}(\omega)\}_{l=1}^{N}$, are independent across technology sources, varieties and sectors with cumulative distribution function

$$\Pr[A_{i1,s} \leq a_{i1,s}, \ldots, A_{iN,s} \leq a_{iN,s}] = \exp\left\{-\left[\sum_{l}\left(\frac{a_{il,s}}{T_{il,s}}\right)^{-\frac{\theta_s}{1-\rho}}\right]^{1-\rho}\right\},\ T_{il,s} > 0,\ \theta_s > 0,\ \rho \in [0,1). \tag{2}$$

The parameter $T_{il,s}$ sets the productivity level of technology from i used in l: increasing it scales up productivity for all sector s goods produced with technology i in l. The parameter θ_s determines how much productivity differs across goods within sector s. A low θ_s implies large productivity differences

across goods, while a high θ_s implies more similar productivities. The common parameter ρ governs how strongly a given technology's productivities are correlated across potential production locations; at $\rho = 0$, they are independent.

Production combines labor with intermediate inputs from each sector. Varieties within an input sector are aggregated with constant elasticity of substitution (CES), with $\sigma_s > 1$ denoting the elasticity of substitution across varieties in sector s. The sectoral inputs and labor are combined through a Cobb–Douglas production function. Let $\alpha_{l,\ell k} > 0$ be the labor cost share and $\alpha_{l,sk} \geq 0$ the share of sector-s inputs used to produce sector-k goods, where ℓ denotes labor and $\alpha_{l,\ell k} + \sum_s \alpha_{l,sk} = 1$. Let w_l be the wage in l and $P_{l,s}$ the price of the bundled sector-s inputs. Cost minimization gives the unit cost of production in country l for sector-k goods

$$u_{l,k} = w_l^{\alpha_{l,\ell k}} \prod_s P_{l,s}^{\alpha_{l,sk}}, \tag{3}$$

where

$$P_{n,s} = \left(\int_0^1 p_{n,s}(\omega)^{1-\sigma_s} \, d\omega \right)^{\frac{1}{1-\sigma_s}}.$$

Impose $\theta_s > \sigma_s - 1$ so that the price index is finite, and let $\tilde{\Gamma}_s \equiv \Gamma(1 - (\sigma_s - 1)/\theta_s)^{1/(1-\sigma_s)}$, where Γ is the Gamma function. Given the productivity distribution (2), applying the law of large numbers across goods provides a closed-form solution for the sectoral price index:

$$P_{n,s} = \tilde{\Gamma}_s \left\{ \sum_i \left[\sum_l \left(\frac{\gamma_{il,s} \tau_{ln,s} (1 + t_{ln,s}) u_{l,s}}{T_{il,s}} \right)^{-\frac{\theta_s}{1-\rho}} \right]^{1-\rho} \right\}^{-1/\theta_s}. \tag{4}$$

Equations (3) and (4) jointly determine production costs and sectoral prices given wages and tariffs.

1.2 Household Preferences

Country n has a representative household that also consumes CES bundles of sectoral goods with elasticity σ_s. Utility across sectoral bundles is aggregated using a Cobb–Douglas function with sectoral expenditure shares $\mu_{n,s} > 0$, where $\sum_s \mu_{n,s} = 1$. Let $q_{n,s}(\omega)$ denote its consumption of variety ω in sector s and I_n its income. Its maximized utility U_n is

$$U_n \equiv \max_{\{q_{n,s}(\omega)\}_{s,\omega}} \prod_s \left(\left[\int_0^1 q_{n,s}(\omega)^{\frac{\sigma_s - 1}{\sigma_s}} \, d\omega \right]^{\frac{\sigma_s}{\sigma_s - 1}} \right)^{\mu_{n,s}} \quad \text{s.t.} \quad \sum_s \int_0^1 p_{n,s}(\omega) q_{n,s}(\omega) \, d\omega \leq I_n. \tag{5}$$

Utility maximisation implies households face the same sectoral price index as producers, $P_{n,s}$ in (4), with the household's aggregate consumption price index given by

$$P_n = \prod_s \left(\frac{P_{n,s}}{\mu_{n,s}} \right)^{\mu_{n,s}}. \tag{6}$$

Let X_n^f denote total final-consumption expenditure. The household exhausts its budget, so $X_n^f = I_n$.

1.3 Expenditure Shares

Let $X_{n,s}$ denote tariff-inclusive expenditure in country n on sector-s goods, which includes both final and intermediate purchases. Let $X_{iln,s}$ denote expenditure on goods whose technology is sourced from i and whose production takes place in l. Define the trilateral expenditure share $\lambda_{iln,s} \equiv X_{iln,s}/X_{n,s}$. The multivariate Fréchet structure combined with household utility maximisation and firm profit maximisation yields

$$\lambda_{iln,s} = \frac{(\tilde{\gamma}_{il,s}\tilde{\tau}_{ln,s}(1+t_{ln,s}))^{-\frac{\theta_s}{1-\rho}}}{\sum_m (\tilde{\gamma}_{im,s}\tilde{\tau}_{mn,s}(1+t_{mn,s}))^{-\frac{\theta_s}{1-\rho}}} \times \frac{\left[\sum_m (\tilde{\gamma}_{im,s}\tilde{\tau}_{mn,s}(1+t_{mn,s}))^{-\frac{\theta_s}{1-\rho}}\right]^{1-\rho}}{\sum_j \left[\sum_m (\tilde{\gamma}_{jm,s}\tilde{\tau}_{mn,s}(1+t_{mn,s}))^{-\frac{\theta_s}{1-\rho}}\right]^{1-\rho}}, \tag{7}$$

where $\tilde{\gamma}_{il,s} \equiv \gamma_{il,s}/T_{il,s}$ is the technology-transfer cost relative to the technology scale, and $\tilde{\tau}_{ln,s} \equiv \tau_{ln,s}u_{l,s}$ combines the unit production and export costs.

Conditional on technology source i, the ratio of country n's expenditure on goods produced in l to expenditure on goods produced domestically is

$$\frac{\lambda_{iln,s}}{\lambda_{inn,s}} = \left[(1+t_{ln,s})\frac{\tilde{\gamma}_{il,s}}{\tilde{\gamma}_{in,s}}\frac{\tilde{\tau}_{ln,s}}{\tilde{\tau}_{nn,s}}\right]^{-\frac{\theta_s}{1-\rho}}. \tag{8}$$

A lower relative cost of using source i's technology in l raises this ratio. For $l, l' \neq n$, its response to the tariff-inclusive cost of production in l' is

$$\frac{\partial \ln(\lambda_{iln,s}/\lambda_{inn,s})}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = -\frac{\theta_s}{1-\rho}\mathbf{1}\{l = l'\}. \tag{9}$$

Conditional on the technology source, this is the constant-elasticity gravity form in Arkolakis, Costinot, and Rodríguez-Clare (2012): $\theta_s/(1-\rho)$ governs the response to a cost change in l, while a cost change in another foreign location has no direct effect on the ratio. These are partial responses, holding wages, sectoral prices, and costs in other locations fixed.

Summing over technology sources gives the bilateral expenditure share

$$\lambda_{ln,s} \equiv \sum_i \lambda_{iln,s}. \tag{10}$$

The ratio of this share to the domestic share separates costs common to all technology sources from their different advantages in producing in l rather than n:

$$\frac{\lambda_{ln,s}}{\lambda_{nn,s}} = \left[(1+t_{ln,s})\frac{\tilde{\tau}_{ln,s}}{\tilde{\tau}_{nn,s}}\right]^{-\frac{\theta_s}{1-\rho}} \sum_i \frac{\lambda_{inn,s}}{\lambda_{nn,s}}\left(\frac{\tilde{\gamma}_{il,s}}{\tilde{\gamma}_{in,s}}\right)^{-\frac{\theta_s}{1-\rho}}. \tag{11}$$

Because technology sources differ in their relative productivity and multinational-production costs across locations, the bilateral share depends on how technology is allocated across production locations as well as

on common production and export costs. Thus, while bilateral shares reflect these fundamentals, they do not reveal the trilateral allocation that determines how those shares respond to a cost change. For $l, l' \neq n$, differentiating the ratio gives the aggregate trade elasticity:

$$\begin{aligned}\frac{\partial \ln(\lambda_{ln,s}/\lambda_{nn,s})}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = & -\frac{\theta_s}{1-\rho}\mathbf{1}\{l=l'\} \\ & + \frac{\rho\theta_s}{1-\rho}\sum_i \frac{\lambda_{il'n,s}}{\sum_m \lambda_{imn,s}}\left(\frac{\lambda_{iln,s}}{\lambda_{ln,s}} - \frac{\lambda_{inn,s}}{\lambda_{nn,s}}\right).\end{aligned} \tag{12}$$

A higher cost in l' directly reduces purchases from l when $l = l'$, but it can also shift where technology sources produce. This changes the mix of technologies embodied in goods from l relative to domestic goods, allowing the expenditure ratio to respond even when $l \neq l'$. How much it responds, and in which direction, depends on whether sources using l' are more important in goods from l or in domestic purchases. Hence the same bilateral shares can imply different aggregate trade elasticities if their underlying trilateral allocations differ.

1.4 General Equilibrium

Tariff revenue TR_n is rebated to the representative household as a lump sum:

$$\mathrm{TR}_n = \sum_s \sum_l \frac{t_{ln,s}}{1+t_{ln,s}} \lambda_{ln,s} X_{n,s}. \tag{13}$$

Households own domestic labor and perfect competition implies zero profits. Final expenditure therefore satisfies

$$X_n^f = I_n = w_n L_n + \mathrm{TR}_n. \tag{14}$$

Let $Y_{l,s}$ denote sector-s gross output in production location l, valued at producer prices. Total sectoral expenditure combines final and intermediate demand:

$$X_{n,s} = \mu_{n,s} X_n^f + \sum_k \alpha_{n,sk} Y_{n,k}. \tag{15}$$

Here $\alpha_{n,sk} Y_{n,k}$ is sector-k producers' expenditure on sector-s inputs. Goods-market clearing requires

$$Y_{l,s} = \sum_n \frac{\lambda_{ln,s} X_{n,s}}{1+t_{ln,s}} \qquad \text{for every } l, s. \tag{16}$$

Labor-market clearing equates labor income to the labor share of producer revenue,

$$w_l L_l = \sum_s \alpha_{l,\ell s} Y_{l,s} \qquad \text{for every } l. \tag{17}$$

Zero profits and equations (13)–(17) imply balanced trade at producer prices.

Nominal variables are homogeneous of degree one. I use world labor income as the numeraire by imposing

$$\sum_{n} w_n L_n = 1. \tag{18}$$

For a fixed tariff schedule $\boldsymbol{t} = \{t_{ln,s}\}_{l,n,s}$ with nonnegative import tariffs and $t_{nn,s} = 0$, a competitive equilibrium consists of wages, sectoral prices, trilateral and bilateral expenditure shares, gross outputs, gross expenditures, final expenditures, and tariff revenues satisfying the unit-cost and price equations (3) and (4), the share equations (7)–(10), the budget and market-clearing equations (13)–(17), and the normalization (18).

2 Optimal Tariffs in the Two-Country One-Sector Model

I specialize the model in Section 1 to two countries, Home (h) and Foreign (f), one sector, and production using labor alone. Thus $u_l = w_l$, and sector subscripts can be suppressed. Let $\bar{w} \equiv w_h/w_f$. Foreign sets $t_{hf} = 0$, while Home chooses $t_{fh} \geq 0$ to maximize its household's real income:

$$\max_{t_{fh} \geq 0} \quad U_h = \frac{w_h L_h}{P_h} \frac{1 + t_{fh}}{1 + t_{fh}\lambda_{hh}}. \tag{19}$$

The wages, prices, and expenditure shares in this objective must satisfy the competitive-equilibrium conditions of Section 1 under the chosen tariff. Appendix B states the specialization and derives the reduced equilibrium.

The two-country restriction isolates the wage and production-location responses that shape the optimal tariff and yields a sharp optimality condition. Even in this setting, the optimal tariff is defined only implicitly: changing the tariff changes wages, prices, and sourcing, so its level must be determined jointly with the equilibrium. I compute tariff levels in the quantitative model, which allows for many countries, sectors, and input–output linkages.

2.1 The Optimal Tariff and the Export-Supply Elasticity

Proposition 1 derives Home's optimal tariff and shows that modelling multinational production does not change the terms of trade motivation for tariffs. Rather, it changes how effective tariffs are at changing the terms of trade that Home faces. To understand this, I will work through how a tariff affects Home's welfare in equilibrium.

A tariff raises the relative cost of Foreign goods in Home and shifts expenditure toward Home production. At fixed relative wages, the increase in Home's domestic expenditure share is measured by

$$\eta^M \equiv \frac{\partial \ln \lambda_{hh}}{\partial \ln[(1 + t_{fh})(w_f/w_h)]}. \tag{20}$$

The fall in demand for Foreign production puts downward pressure on Foreign's wage relative to Home's. The response of Foreign export supply to this change in relative wages is measured by

$$\eta^X \equiv \frac{\partial \ln \lambda_{hf}}{\partial \ln(w_f/w_h)}. \tag{21}$$

Trade balance makes $L_f \lambda_{hf}$ the Foreign labor used to supply Home, so η^X is the export-supply elasticity faced by Home. In equilibrium, the benefit from a tariff depends on how much Home's relative wage rises, and hence how much the price of Foreign exports falls relative to Home's goods. Let $g \equiv d \ln \bar{w}/d \ln(1 + t_{fh})$ denote the equilibrium relative-wage response to the tariff, which depends on both η^M and η^X. The larger this change, the greater the terms-of-trade gain.

The tariff also creates a distortion. Consumers face an import price above its resource cost and do not internalize the tariff revenue rebated to the household. They therefore import fewer Foreign goods. The

marginal terms-of-trade gain and distortion together determine the welfare effect of the tariff:

$$\frac{d\ln U_h}{d\ln(1+t_{fh})} = \underbrace{\frac{\lambda_{fh} g}{1+t_{fh}\lambda_{hh}}}_{\text{terms-of-trade gain}} - \underbrace{\frac{t_{fh}\lambda_{hh}(\lambda_{fh}+\eta^M)(1-g)}{1+t_{fh}\lambda_{hh}}}_{\text{tariff distortion}}. \tag{22}$$

An interior optimum equates the two terms.

Proposition 1 (The optimal tariff and the export-supply channel)
Any interior optimal import tariff satisfies

$$t^*_{fh} = \left.\frac{1}{\eta^X}\right|_{t_{fh}=t^*_{fh}}. \tag{23}$$

With multinational production, the export-supply elasticity at any equilibrium is

$$\eta^X = \frac{\theta}{1-\rho}\lambda_{ff} - \frac{\rho\theta}{1-\rho}\left[\sum_{i\in\{h,f\}}\frac{\lambda_{ihf}}{\lambda_{hf}}\frac{\lambda_{ihf}}{\lambda_{ihf}+\lambda_{iff}} - \lambda_{hf}\right]. \tag{24}$$

All shares and elasticities in the optimality condition are evaluated at the equilibrium induced by the optimal tariff.

Multinational production does not change the traditional terms-of-trade motive for a tariff. It adds relocation of production to the quantity and wage responses to a tariff. Although g depends on both η^M and η^X, Proposition 1 shows that the export-supply elasticity η^X determines the optimal tariff.

2.2 Lowering Multinational-Production Costs

To understand how multinational production changes the gains from a tariff, Proposition 1 directs us to the export-supply elasticity. We first compare an economy with prohibitive multinational-production costs to one with finite costs. As we lower these costs, we adjust technology scales and trade costs so that the two economies have the same bilateral expenditure shares at the comparison equilibrium. The comparison is therefore between the responses of the same trade allocation, with and without the ability to relocate production in the face of a tariff.

Proposition 2 (Changing technology transfer costs)
Start from prohibitive multinational-production costs, $\gamma_{il} \to \infty$ for $i \neq l$, and lower them to finite values. Adjust T_{il} and τ_{ln} so that the two economies have the same bilateral expenditure shares at the comparison equilibrium. Denote the export supply elasticity in the prohibitive multinational-production cost economy by $\eta^{X,\text{nomp}}$. Then

$$\eta^X - \eta^{X,\text{nomp}} = \frac{\rho\theta}{(1-\rho)\lambda_{hf}}\sum_{i\in\{h,f\}}\frac{\lambda_{ihf}\lambda_{iff}}{\lambda_{ihf}+\lambda_{iff}} \geq 0. \tag{25}$$

Finite costs imply positive trilateral shares, so the inequality is strict if $\rho > 0$. At the common allocation, allowing multinational production weakly raises the export-supply elasticity.

When multinational production is prohibited, a cost change can shift expenditure between goods produced

with Home and Foreign technologies, but cannot change where either technology is used. Finite costs add this location choice. This gives technology sources another way to respond to a change in relative costs, making export supply more responsive. The size of the increase depends on how readily they can supply the same quantities at the same prices by changing where they produce. The higher export-supply elasticity weakens the terms-of-trade incentive to raise the tariff at the common allocation; the actual wage response also depends on η^M. Matching shares at the comparison equilibrium does not hold them fixed after a tariff change or rank optimal tariffs at the two endogenous optima.

2.3 Modeling Multinational Production at Common Trade Shares

The next question is how modeling multinational production changes the gains from a tariff when both models have the same bilateral trade shares. Unlike the previous comparison, we now compare different productivity structures. Proposition 1 again directs us to their export-supply elasticities.

The trade-only model retains the preferences, labor endowments, trade-cost and tariff structure, and tariff-revenue rebate of the two-country model. It has no separate technology-source choice or multinational-production cost γ_{il}. Instead, productivity follows Eaton and Kortum (2002): each production location has an independent Fréchet draw with shape θ^{trade} and scale T_l^{trade},

$$\Pr[A_l^{\text{trade}}(\omega) \leq a] = \exp\left[-\left(\frac{a}{T_l^{\text{trade}}}\right)^{-\theta^{\text{trade}}}\right].$$

In the trade-only model, the export supply elasticity is given by $\eta^{X,\text{trade}} = \theta^{\text{trade}}\lambda_{ff}$.

The trade-only model records where goods are produced, whereas the multinational-production model also records which technology source produces them. For example, Home technology can supply Foreign by producing in Home, measured by λ_{hhf}, or by producing in Foreign, measured by λ_{hff}. Its relative production odds satisfy

$$\ln\left(\frac{\lambda_{hhf}}{\lambda_{hff}}\right) = \frac{\theta}{1-\rho}\ln\left[\frac{T_{hh}}{T_{hf}}\frac{\gamma_{hf}}{\gamma_{hh}}\frac{1}{\tau_{hf}}\frac{w_f}{w_h}\right]. \tag{26}$$

More Home production therefore reveals a net advantage for Home technology in producing there, reflecting relative productivity, multinational-production costs, and trade costs. Bilateral shares alone conceal this source-specific advantage.

The net advantage in (26) can differ across technology sources. To summarize these differences, note that the expenditure-weighted mean fraction of Foreign purchases produced in Home is λ_{hf}. The dispersion of each source's Home-production fraction around this mean is

$$V_f \equiv \sum_{i\in\{h,f\}}(\lambda_{ihf}+\lambda_{iff})\left(\frac{\lambda_{ihf}}{\lambda_{ihf}+\lambda_{iff}}-\lambda_{hf}\right)^2. \tag{27}$$

Because each source's fraction reflects its net relative advantage, a larger V_f means that the sources differ more in where they are best placed to produce. The variance is zero when both sources use the same production mix and approaches $\lambda_{hf}\lambda_{ff}$ as each source concentrates production in one location. Rewriting

(24) gives

$$\eta^X = \frac{\theta}{1-\rho}\lambda_{ff} - \frac{\rho\theta}{1-\rho}\frac{V_f}{\lambda_{hf}}. \tag{28}$$

At given bilateral shares and parameters, greater dispersion lowers the export-supply elasticity: a technology source whose net advantage strongly favors one location changes its production mix less for a given change in relative costs.

At common bilateral shares, $\eta^{X,\text{trade}} = \theta^{\text{trade}}\lambda_{ff}$ differs from (28) through both the shape parameter and V_f. I first compare two links between the shape parameters, then give the general condition.

Proposition 3 (Modeling multinational production with $\theta/(1-\rho) = \theta^{\text{trade}}$)
For admissible shape parameters satisfying the finite-price-index conditions in both models, adjust technology scales and trade costs so that the trade-only model and multinational-production model have the same bilateral shares at the comparison equilibrium. Set

$$\theta^{\text{trade}} = \frac{\theta}{1-\rho}. \tag{29}$$

Then

$$\eta^X - \eta^{X,\text{trade}} = -\frac{\rho\theta}{1-\rho}\frac{V_f}{\lambda_{hf}} \leq 0. \tag{30}$$

The inequality is strict if $\rho > 0$ and the two technology sources use different production mixes, resulting in $V_f > 0$.

This comparison matches the trade-only cost sensitivity of production to the response within a given technology source in the multinational production model, given by (9). When the sources have different net advantages, as revealed by their production odds, they sort across locations. The trade-only model matches the bilateral shares but cannot account for that sorting. It therefore predicts a larger export-supply response than the multinational-production model when $\rho > 0$ and $V_f > 0$, weakening the terms-of-trade incentive to raise the tariff relative to the model with multinational production.

Proposition 4 (Modeling multinational production with $\theta^{\text{trade}} = \theta$)
For admissible shape parameters satisfying the finite-price-index conditions in both models, adjust technology scales and trade costs so that the trade-only model and multinational-production model have the same bilateral shares at the comparison equilibrium. Set

$$\theta^{\text{trade}} = \theta. \tag{31}$$

$$\eta^X - \eta^{X,\text{trade}} = \frac{\rho\theta}{1-\rho}\left(\lambda_{ff} - \frac{V_f}{\lambda_{hf}}\right) \geq 0. \tag{32}$$

With finite multinational-production costs, the inequality is strict if $\rho > 0$.

This comparison holds marginal productivity dispersion fixed rather than matching the sensitivity of production-location choices. For $\rho > 0$, the cost sensitivity of production, conditional on a technology source, $\theta/(1-\rho)$ exceeds θ^{trade}. Allowing a technology source to move production between countries therefore adds responsiveness to a cost change. Differences in net advantages across sources dampen that response through V_f, but the variance bound ensures that they cannot more than offset the additional cost

sensitivity. The export-supply elasticity is consequently weakly higher than in the trade-only model, as in the prohibitive-to-finite-cost comparison.

Proposition 5 (Modeling multinational production)
For admissible shape parameters satisfying the finite-price-index conditions in both models, adjust technology scales and trade costs so that the trade-only model and multinational-production model have the same bilateral shares at the comparison equilibrium. Fix a V_f that satisfies this equilibrium in the model with multinational production.

$$\eta^X \gtreqless \eta^{X,\text{trade}} \quad \Longleftrightarrow \quad \frac{\theta}{1-\rho}\left(1-\rho\frac{V_f}{\lambda_{hf}\lambda_{ff}}\right) \gtreqless \theta^{\text{trade}}. \tag{33}$$

V_f measures how much source-specific production choices dampen the response implied by the within-source cost sensitivity $\theta/(1-\rho)$. Whether modeling multinational production raises or lowers the export-supply elasticity therefore depends jointly on the relationship between the shape parameters and the source-specific net advantages revealed by production choices. By Proposition 1, a lower elasticity strengthens the terms-of-trade incentive to raise the tariff at the common allocation. These comparisons do not rank optimal tariffs at the two endogenous optima, because a tariff changes wages, bilateral shares, and technology-source composition. The quantitative exercise implements these counterfactuals and solves for each model's optimal tariff and its associated equilibrium, allowing us to assess whether the stronger terms-of-trade motive at the common allocation translates into higher optimal tariffs.

3 Quantitative Optimal Tariffs

I now return to the general model from Section 1. Home chooses import tariffs by production origin and sector to maximize its representative household's welfare, taking other countries' tariffs as given. Let $\boldsymbol{t}_h \equiv \{t_{lh,s} : l \neq h,\ s = 1, \ldots, S\}$ be Home's tariff vector, with each tariff in $[0, \bar{t}]$ for a fixed upper bound $\bar{t} \in (0, 1)$. Let $\bar{\boldsymbol{t}}_{-h}$ collect the fixed tariffs of other countries. Denote the full tariff schedule by $\boldsymbol{t} = (\boldsymbol{t}_h, \bar{\boldsymbol{t}}_{-h})$ and let $z(\boldsymbol{t})$ denote the competitive equilibrium under that schedule. Home's problem is

$$\max_{\boldsymbol{t}_h \in [0,\bar{t}]^{(N-1)S}} U_h \quad \text{subject to} \quad z(\boldsymbol{t}) \text{ being a competitive equilibrium under } \boldsymbol{t}. \tag{34}$$

The equilibrium conditions are those in Section 1; Appendix A.5 states the full constrained problem. A tariff changes Home's prices and revenue as well as wages, production locations, and expenditure throughout the international production network. I first explain how I solve for the optimal tariff vector, then discuss why varying tariffs by exporter and sector can matter.

3.1 Solving for Optimal Tariffs

Home's optimal tariff problem in (34) is written as a nested problem in the spirit of Rust (1987). The stacked vectors of variables that define the model equilibrium are

$$\begin{array}{lcllcl} \boldsymbol{w} & \equiv & \{w_n\}_n \in \mathbb{R}^N_{++}, & \boldsymbol{P} & \equiv & \{P_{n,s}\}_{n,s} \in \mathbb{R}^{NS}_{++}, \\ \boldsymbol{\lambda} & \equiv & \{\lambda_{ln,s}\}_{l,n,s} \in \mathbb{R}^{N^2S}_{++}, & \boldsymbol{Y} & \equiv & \{Y_{n,s}\}_{n,s} \in \mathbb{R}^{NS}_{++}, \\ \boldsymbol{X} & \equiv & \{X_{n,s}\}_{n,s} \in \mathbb{R}^{NS}_{++} & & & \end{array}$$

where each component of $\boldsymbol{\lambda}$ is obtained by summing over technology sources, $\lambda_{ln,s} = \sum_i \lambda_{iln,s}$. The competitive equilibrium under tariff vector $\boldsymbol{t}$ is then

$$z(\boldsymbol{t}) \equiv (\boldsymbol{w}(\boldsymbol{t}), \boldsymbol{P}(\boldsymbol{t}), \boldsymbol{\lambda}(\boldsymbol{t}), \boldsymbol{Y}(\boldsymbol{t}), \boldsymbol{X}(\boldsymbol{t})).$$

I solve the welfare problem using the log objective

$$\tilde{U}_h(\boldsymbol{t}_h; \bar{\boldsymbol{t}}_{-h}) \equiv \log U_h(z(\boldsymbol{t}), \boldsymbol{t})\,.$$

The inner problem in (34) maps each candidate $\boldsymbol{t}_h$ into $z(\boldsymbol{t})$; the outer problem searches across candidate tariffs to find the tariff vector that maximizes Home's welfare in equilibrium. As emphasized by Wang et al. (2025), the interaction between the inner and outer problems creates the computational challenge of solving for Home's best response in quantitative trade models. The outer problem is high-dimensional, and because $z(\boldsymbol{t})$ has no closed-form solution, evaluating welfare at any candidate tariff vector requires solving the full general-equilibrium system. I exploit the structure of the model to reduce the computational burden by computing the total derivative $\nabla_{\boldsymbol{t}_h} \tilde{U}_h$—which captures the effect of a tariff change on Home's welfare through the induced equilibrium allocation—as a function of partial derivatives for which analytical solutions are available. A bound-constrained quasi-Newton algorithm then solves for Home's optimal unilateral tariffs.

To see how the derivative of equilibrium welfare is obtained, note that the inner equilibrium problem has the following block structure. Given $(\boldsymbol{w}, \boldsymbol{t})$, equations (3) and (4) jointly determine $\boldsymbol{P}$; the Fréchet allocation formulas (7) and (10) then determine $\boldsymbol{\lambda}$; equations (A.20) and (A.21) determine $\boldsymbol{Y}$ and $\boldsymbol{X}$ through

linear systems. Wages close the model. In particular, following Alvarez and Lucas (2007), define country n's labor-market residual by

$$Z_n(\boldsymbol{w}, \boldsymbol{t}) \equiv \sum_s \alpha_{n,\ell s} Y_{n,s}(\boldsymbol{w}, \boldsymbol{t}) - w_n L_n.$$

By Walras' law, one of the N labor-market equations is redundant. Let $\boldsymbol{F}(\boldsymbol{w}, \boldsymbol{t}) = \mathbf{0}$ stack any $N-1$ independent residuals $Z_n = 0$ together with the normalization $\sum_n w_n L_n = 1$. Thus, conditional on candidate wages and tariffs, the remaining equilibrium objects are constructed as

$$(\boldsymbol{w}, \boldsymbol{t}) \longmapsto \boldsymbol{P}(\boldsymbol{w}, \boldsymbol{t}) \longmapsto \boldsymbol{\lambda}(\boldsymbol{w}, \boldsymbol{P}, \boldsymbol{t}) \longmapsto \boldsymbol{Y}(\boldsymbol{w}, \boldsymbol{\lambda}, \boldsymbol{t}) \longmapsto \boldsymbol{X}(\boldsymbol{w}, \boldsymbol{\lambda}, \boldsymbol{Y}, \boldsymbol{t}),$$

and the equilibrium wage vector $\boldsymbol{w}(\boldsymbol{t})$ solves $\boldsymbol{F}(\boldsymbol{w}(\boldsymbol{t}), \boldsymbol{t}) = \mathbf{0}$.

Substituting the price, share, output, and expenditure blocks into welfare while temporarily holding wages fixed gives the reduced objective

$$\phi(\boldsymbol{w}, \boldsymbol{t}) \equiv \log U_h\Big(\boldsymbol{w}, \boldsymbol{P}(\boldsymbol{w}, \boldsymbol{t}), \boldsymbol{\lambda}(\boldsymbol{w}, \boldsymbol{t}), \boldsymbol{Y}(\boldsymbol{w}, \boldsymbol{t}), \boldsymbol{X}(\boldsymbol{w}, \boldsymbol{t}); \boldsymbol{t}\Big).$$

Let $J_{\boldsymbol{y}}\boldsymbol{x}$ denote the Jacobian of $\boldsymbol{x}$ with respect to $\boldsymbol{y}$, and take gradients to be column vectors. The chain rule and the implicit function theorem give

$$\begin{aligned} \nabla_{\boldsymbol{t}_h} \tilde{U}_h &= \nabla_{\boldsymbol{t}_h} \phi + \left[J_{\boldsymbol{t}_h} \boldsymbol{w}(\boldsymbol{t})\right]^\top \nabla_{\boldsymbol{w}} \phi, \\ J_{\boldsymbol{t}_h} \boldsymbol{w}(\boldsymbol{t}) &= -\left[J_{\boldsymbol{w}} \boldsymbol{F}\right]^{-1} J_{\boldsymbol{t}_h} \boldsymbol{F}, \end{aligned}$$

with all derivatives evaluated at the candidate equilibrium. Combining these expressions yields the total derivative

$$\nabla_{\boldsymbol{t}_h} \tilde{U}_h = \nabla_{\boldsymbol{t}_h} \phi - \left[J_{\boldsymbol{t}_h} \boldsymbol{F}\right]^\top \left[J_{\boldsymbol{w}} \boldsymbol{F}\right]^{-\top} \nabla_{\boldsymbol{w}} \phi.$$

The partial gradients of ϕ and the derivatives of $\boldsymbol{F}$ incorporate the responses of prices, trilateral and bilateral shares, gross output, and gross expenditure at fixed wages. The high dimension of $\boldsymbol{t}_h$ makes forward propagation of the full policy Jacobian computationally taxing because it must carry one sensitivity column for every tariff. Instead, I apply the adjoint method as an exercise in collecting coefficients. Starting from the total differential of welfare, I substitute the differentials of gross expenditure, gross output, trade shares, prices, and wages while collecting the coefficients multiplying the remaining endogenous differentials. Once all endogenous differentials have been substituted out, the coefficients on $d\boldsymbol{t}_h$ are exactly $\nabla_{\boldsymbol{t}_h} \tilde{U}_h$. This procedure computes the welfare gradient without forming the full policy Jacobian of each intermediate equilibrium object. Appendix E derives the partial derivatives and details the adjoint calculation.

3.2 Country–Sector Tariffs

Section 2 covers the main intuition for how multinational production changes the optimal tariff when Home has one import tariff. However, differentiating tariffs by exporter and sector can result in additional welfare gains relative to a uniform tariff. The differentiated schedule can exploit both sectoral production-network effects and origin-specific wage responses.

To understand sector and origin targeting, write the welfare response to a tariff as a fixed-wage effect and

an effect through equilibrium wages, as in Section 3.1:

$$\frac{d\ln U_h}{d\ln(1+t_{lh,k})} = \left.\frac{\partial \ln U_h}{\partial \ln(1+t_{lh,k})}\right|_{\boldsymbol{w}} + \sum_j \left.\frac{\partial \ln U_h}{\partial \ln w_j}\right|_{\boldsymbol{t}} \frac{d\ln w_j}{d\ln(1+t_{lh,k})}. \tag{35}$$

In the first term, wages are held fixed while prices, expenditure shares, and the remaining equilibrium objects adjust to the tariff. The partial derivative with respect to wage j holds tariffs and other wages fixed while allowing the remaining equilibrium objects to adjust. Multiplying this derivative by the equilibrium wage response gives wage j's contribution to the welfare effect. Following the block structure in Section 3.1, I first trace how tariffs affect prices at fixed wages, then allow wages to adjust. Together with the sourcing responses, these price changes determine expenditure and welfare.

The sectoral motive follows from the existing literature. In the two-country Ricardian environment of Costinot et al. (2015), the import-tax component is uniform across goods even in the constrained problem without export-tax instruments, while comparative advantage shapes differentiated export wedges when those instruments are available. With intermediate inputs and a tariff-only instrument, however, import tariffs propagate through downstream production costs and can shift part of their incidence to foreign consumers. This creates the sector-specific component characterized by Beshkar and Lashkaripour (2020).

To see how input–output linkages generate sectoral differentiation, define

$$\Psi \equiv (I-\mathbf{O})^{-1} = I + \mathbf{O} + \mathbf{O}^2 + \mathbf{O}^3 + \cdots, \qquad \mathbf{O}_{(n,s),(l,k)} \equiv \lambda_{ln,s}\alpha_{l,ks}.$$

The matrix Ψ summarizes direct and downstream propagation through the global production network. $\psi_{(n,s),(d,k)} \equiv [\Psi]_{(n,s),(d,k)}$ denotes the propagation coefficient from a direct price shock at destination–sector node (d,k) to the price at node (n,s). It includes the direct effect when $(n,s)=(d,k)$ and, through input linkages, the effects on $P_{n,s}$ when $(n,s)\neq(d,k)$. The fixed-wage response to a tariff levied by h on sector-k goods produced in l is

$$\left.\frac{\partial \ln P_{n,s}}{\partial \ln(1+t_{lh,k})}\right|_{\boldsymbol{w}} = \psi_{(n,s),(h,k)}\lambda_{lh,k}. \tag{36}$$

The bilateral share $\lambda_{lh,k}$, evaluated at the initial equilibrium, measures the direct effect on $P_{h,k}$: a tariff on goods produced in l matters more for this price when those goods account for more of expenditure. The rise in $P_{h,k}$ then raises costs in sectors that use Home's sector-k bundle as an input, with further effects on sectoral prices across countries. The coefficient $\psi_{(n,s),(h,k)}$ summarizes how this initial price change reaches (n,s) through direct and downstream input use. Differences across sectors in this propagation create the production-network motive for sector targeting even before accounting for equilibrium wage responses. Appendix A.3 derives (36) from the sectoral price index.

The literature provides less direct guidance on origin differentiation. The quantitative tariff-only exercise in Beshkar and Lashkaripour (2020) aggregates foreign countries when computing sectoral tariffs. Lashkaripour (2021) allows bilateral-industry tariffs. However, its main sufficient-statistics formulas approximate away the welfare effects that operate through relative wages within the rest of the world, and its input–output specification prevents import tariffs from propagating through the production network. An approximation-free exercise without input–output linkages finds modest partner variation in a smaller Ricardian calibration. The exercise in this paper studies origin differentiation with multinational production,

multiple sectors, input–output linkages, and an unrestricted vector of foreign wage responses.

For a Home tariff on imports from country l in sector k, define the wage-response vector

$$\boldsymbol{g}^{l,k} \equiv \frac{d\ln \boldsymbol{w}}{d\ln(1+t_{lh,k})},$$

where $g_{l'}^{l,k}$ denotes the wage response of country l'. The bilateral terms-of-trade channel is

$$g_h^{l,k} - g_l^{l,k} = \frac{d\ln(w_h/w_l)}{d\ln(1+t_{lh,k})}. \tag{37}$$

When the tariff lowers w_l relative to w_h, imports from l become cheaper relative to Home's income. This bilateral terms-of-trade gain is one part of the wage channel in (35). The tariff also changes wages elsewhere as prices, expenditure, and labor demand adjust across countries; these wage changes affect Home's welfare in addition to the fixed-wage effects.

Define $\eta_{n,s}^{ll'}$ as the elasticity of $\lambda_{ln,s}$ to the tariff-inclusive unit cost of goods produced in l', holding other production locations' costs fixed:

$$\eta_{n,s}^{ll'} \equiv \frac{\partial \ln \lambda_{ln,s}}{\partial \ln[(1+t_{l'n,s})u_{l',s}]}.$$

Appendix A.2 derives the expression for this elasticity. Then, a change in $w_{l'}$ results in

$$\left.\frac{\partial \ln \lambda_{ln,s}}{\partial \ln w_{l'}}\right|_{\boldsymbol{P},\boldsymbol{t}} = \alpha_{l',\ell s}\eta_{n,s}^{ll'}. \tag{38}$$

Setting $l' = l$ shows that a change in the targeted exporter's wage reallocates expenditure and labor demand between l and alternative suppliers in every market. Holding tariffs and other wages fixed, the price response to a wage change allows sectoral prices and sourcing shares to adjust through input–output linkages:

$$\frac{\partial \ln P_{n,s}}{\partial \ln w_{l'}} = \sum_{d,k} \alpha_{l',\ell k}\lambda_{l'd,k}\psi_{(n,s),(d,k)}. \tag{39}$$

Here, $\lambda_{l'd,k}\alpha_{l',\ell k}$ measures the direct effect of $w_{l'}$ on price $P_{d,k}$, and $\psi_{(n,s),(d,k)}$ carries that effect to (n,s). Combining (36) and (39), the price response to a tariff, including the general equilibrium change in wages, is

$$\begin{aligned}\frac{d\ln P_{n,s}}{d\ln(1+t_{lh,k})} &= \left.\frac{\partial \ln P_{n,s}}{\partial \ln(1+t_{lh,k})}\right|_{\boldsymbol{w}} + \sum_{l'} \frac{\partial \ln P_{n,s}}{\partial \ln w_{l'}} g_{l'}^{l,k} \\ &= \psi_{(n,s),(h,k)}\lambda_{lh,k} + \sum_{l'}\left(\sum_{n',k'} \alpha_{l',\ell k'}\psi_{(n,s),(n',k')}\lambda_{l'n',k'}\right) g_{l'}^{l,k}.\end{aligned} \tag{40}$$

Equations (36) and (40) show why a tariff's effect can differ across origins within the same sector. The direct change in Home's sector-k price depends on the import share from l and, through input linkages, affects prices in other sectors. These price and sourcing changes are part of the fixed-wage welfare effect in (35). In equilibrium, the tariff also changes wages across countries, which further changes prices and sourcing and enters the wage term of that equation. Thus, countries with similar Home import shares

need not generate the same welfare response if their tariff-induced wage changes differ.

For comparison, consider a hypothetical restriction under which all foreign wages move together:

$$g_n^{l,k} - g_{n'}^{l,k} = 0 \qquad \text{for all } n, n' \neq h.$$

This restriction rules out tariff-induced changes in wages relative to one another among foreign countries. A tariff can still change the Home wage relative to the common foreign wage, but it cannot lower w_l relative to the wages of alternative foreign suppliers.

This hypothetical restriction differs from the welfare approximation used in the main sufficient-statistics formulas of Lashkaripour (2021), which omits the welfare contribution of relative foreign wage changes without requiring equal foreign wage responses. Here I solve the multinational-production input–output tariff problem while retaining the unrestricted wage vector and its effects on sourcing, prices, and welfare.

Sector k therefore determines where the tariff enters the production network, while origin l determines Home's exposure to the targeted supplier and the wage response across countries. A differentiated tariff can take both margins into account. Multinational production changes these responses without introducing a new welfare motive because it changes how expenditure shares respond to costs, as in Section 2. Technology-source sorting affects how strongly a tariff reduces labor demand and wages in l, while sources' alternative production locations determine where production and labor demand are redirected. Multinational production therefore changes the expenditure share elasticities in (38) and hence the equilibrium wage-response vector $\boldsymbol{g}^{l,k}$, with effects that can vary across origin–sector pairs.

4 Calibration

The focus of the quantitative exercise is how the equilibrium changes when tariffs change. I therefore solve the model in exact hats, following Dekle, Eaton, and Kortum (2008) and Caliendo and Parro (2015). For any endogenous variable x, a hat denotes its counterfactual value relative to its baseline equilibrium value, $\hat{x} \equiv x'/x$, where x is its equilibrium value under the baseline tariff vector $\boldsymbol{t}$ and x' is its equilibrium value under the counterfactual tariff vector $\boldsymbol{t}'$. The system in (E.2)–(E.11) of Appendix E determines the counterfactual equilibrium given these tariff vectors, the Fréchet parameters $\{\theta_s\}_{s=1}^{S}$ and ρ, labor shares $\alpha_{n,\ell s}$, intermediate-input shares $\alpha_{n,ks}$, final-consumption shares $\mu_{n,s}$, and baseline equilibrium expenditure shares $\lambda_{iln,s}$, gross output $Y_{n,s}$, and gross expenditure $X_{n,s}$.

Solving the model in exact hats reduces the number of parameters that must be calibrated. Because technology parameters $T_{il,s}$ and iceberg costs $\tau_{ln,s}$ and $\gamma_{il,s}$ remain unchanged in a tariff counterfactual, their role in the counterfactual equilibrium can be recovered from baseline expenditure shares. For given θ_s and ρ, matching these shares implicitly calibrates the combinations of technology and cost parameters that determine sourcing, without separately identifying each parameter. The model therefore reproduces the baseline allocation by construction, while its adjustment mechanisms determine how the equilibrium changes with tariffs.

I first describe how the data determine labor shares, intermediate-input shares, and final-consumption shares. For given θ_s and ρ, I construct expenditure shares, gross output, and gross expenditure from the data. The resulting zero-tariff equilibrium is the baseline equilibrium throughout the paper unless otherwise stated. I use empirical trade elasticity estimates and a restriction on ρ to assign values to θ_s. I then discuss the calibration of the quantitative economies that implement the multi-country, multisector versions of the counterfactuals discussed in Sections 2.2 and 2.3.

4.1 Data

I use the inter-country input–output and ownership tables from the OECD Analytical Activities of Multinational Enterprises (AAMNE) database (Cadestin et al. 2018). The input–output tables report gross output, value added, intermediate use, final use, and bilateral sales by production and destination country and sector. The ownership tables report output by ownership country, production country, and sector. I aggregate these data to 45 countries and 33 sectors, as reported in Tables D.2 and D.1. The data are reported at basic prices, which exclude taxes less subsidies on products. Trade and transport margins are allocated to the corresponding service sectors. These flows therefore exclude import tariffs. I use data from 2020, the latest year provided in the 2024 release, to calibrate the labor, intermediate-input, and final-consumption shares and construct the baseline equilibrium values of $\lambda_{iln,s}$, $Y_{n,s}$, and $X_{n,s}$. Table 1 summarizes the mapping.

I make three adjustments to align the accounts with the model. First, because labor is the only factor of production, I assign value added to labor income. When reported value added is nonpositive, I replace its share in gross output using positive shares from the same country and sector in adjacent years, interpolating across consecutive nonpositive observations when necessary. I then adjust intermediate purchases so that value added and intermediate-input spending continue to sum to gross output, while preserving the composition of those purchases across sectors and origins. Second, because the model has no inventory accumulation, I allocate inventory changes to final consumption and reconcile final sales with the adjusted intermediate purchases, while preserving the distribution of final sales across destinations. Third, I replace

zero trade flows and zero ownership-production flows with small positive values, as finite costs and positive technology scales imply positive flows in the model. These replacements preserve total sales from each production country and sector. Appendix D.1 provides the details of the implementation of these adjustments.

Let $\mathrm{VA}_{n,s}$ denote value added, $\mathrm{IO}_{ln,ks}$ purchases of sector-k inputs from l by sector s in n, and $X^f_{ln,s}$ final purchases of sector-s goods produced in l and consumed in n. The cleaned accounts determine the model's cost and final-demand shares:

$$\alpha_{n,\ell s} = \frac{\mathrm{VA}_{n,s}}{Y_{n,s}}, \qquad \alpha_{n,ks} = \frac{\sum_l \mathrm{IO}_{ln,ks}}{Y_{n,s}}, \qquad \mu_{n,s} = \frac{\sum_l X^f_{ln,s}}{\sum_{l,r} X^f_{ln,r}}.$$

Intermediate and final purchases determine bilateral expenditures and expenditure shares:

$$X_{ln,s} = \sum_k \mathrm{IO}_{ln,sk} + X^f_{ln,s}, \qquad X_{n,s} = \sum_l X_{ln,s}, \qquad \lambda_{ln,s} = \frac{X_{ln,s}}{X_{n,s}}.$$

The ownership data provide the additional information needed to recover trilateral expenditure shares. Let $Y^{\mathrm{MP}}_{il,s}$ denote output in country l and sector s produced using technology owned in i. These data do not reveal where this output is sold. I recover the trilateral expenditure shares as the values of $\lambda_{iln,s}$ that satisfy the model's sourcing formula in (7) and the two observed margins:

$$\sum_n \lambda_{iln,s} X_{n,s} = Y^{\mathrm{MP}}_{il,s}, \qquad \sum_i \lambda_{iln,s} X_{n,s} = X_{ln,s}.$$

For given ρ, the separability of source–location and location–destination costs restricts how these two margins can be combined and determines a unique trilateral allocation. Appendix D.2 establishes this uniqueness.

The model imposes trade balance in equilibrium; the data have no such restriction. I remove trade imbalances in the data by solving for a no-transfer equilibrium, following Costinot and Rodríguez-Clare (2014). The resulting gross output $Y^{\mathrm{data}}_{n,s}$, gross expenditure $X^{\mathrm{data}}_{n,s}$, and trilateral expenditure shares $\lambda^{\mathrm{data}}_{iln,s}$ satisfy the equilibrium conditions and are the baseline targets that the model reproduces exactly. Appendix D.3 describes the balancing procedure.

Table 1: Baseline calibration targets

Model object	Construction from the AAMNE data
Gross output, $Y_{l,s}^{\text{data}}$	Reported production-account output, adjusted to satisfy trade balance.
Labor cost share, $\alpha_{l,\ell s}$	Gross value added plus the valuation adjustment, divided by gross output, with nonpositive observations corrected.
Input–output shares, $\alpha_{l,ks}$	Intermediate purchases of sector-k inputs by sector s, divided by gross output.
Final-demand shares, $\mu_{n,s}$	Adjusted final expenditure on sector s as a share of total final expenditure in n.
Bilateral expenditures and shares, $X_{ln,s}^{\text{data}}$ and $\lambda_{ln,s}^{\text{data}}$	Intermediate plus final purchases by production country, destination, and sector, adjusted with the other flows to satisfy trade balance.
Trilateral expenditures and shares, $X_{iln,s}^{\text{data}}$ and $\lambda_{iln,s}^{\text{data}}$	Recovered from ownership-production output $Y_{il,s}^{\text{MP}}$ and bilateral sales using the model's share restrictions, then adjusted with the other flows to satisfy trade balance.

Notes: The superscript data denotes the final model-consistent baseline.

4.2 Calibrating the Fréchet Parameters

The trade elasticity as defined in Arkolakis, Costinot, and Rodríguez-Clare (2012) describes how expenditure on imported goods relative to domestic goods responds to trade costs. In the present model, the trade elasticity is given by (12). For $l, l' \neq n$,

$$\begin{aligned}\varepsilon_{n,s}^{ll'} &\equiv \frac{\partial \ln(X_{ln,s}/X_{nn,s})}{\partial \ln[(1+t_{l'n,s})u_{l',s}]} \\ &= -\frac{\theta_s}{1-\rho}\mathbf{1}\{l=l'\} + \frac{\rho\theta_s}{1-\rho}\sum_i \frac{\lambda_{il'n,s}}{\sum_m \lambda_{imn,s}}\left(\frac{\lambda_{iln,s}}{\lambda_{ln,s}} - \frac{\lambda_{inn,s}}{\lambda_{nn,s}}\right).\end{aligned} \tag{41}$$

The first term is the response of production locations to a change in costs that is common to all technology sources. The second captures how the technology composition of imports relative to domestic goods changes in response to the change in costs. Thus, the trade elasticity is an endogenous object that depends on equilibrium expenditure shares and differs across country pairs.

Arkolakis, Costinot, and Rodríguez-Clare (2012) show that a large class of trade models follows a gravity structure in which bilateral expenditure shares are the product of an exporter-specific component, an importer-specific component, and bilateral trade costs raised to a common exponent. The implied import demand system satisfies independence of irrelevant alternatives. In these models, the common exponent is the trade elasticity: it is determined by model parameters, is independent of expenditure shares, and is constant across country pairs. A gravity regression estimates a constant elasticity across country pairs for the response of bilateral trade flows to trade costs, controlling for country-specific components. This provides an empirical estimate of the trade elasticity. For example, Caliendo and Parro (2015) estimate sector-specific coefficients from trade and tariff variation across country triplets, using ratios that remove exporter, importer, and systematic non-tariff cost components. In their multisector Ricardian model with input–output linkages, the magnitude of each coefficient maps directly to the sector's Fréchet shape parameter.

Multinational production breaks this direct mapping because of the second term in (41) (Ramondo and Rodríguez-Clare 2013; Arkolakis et al. 2018). To calibrate θ and ρ, Arkolakis et al. (2018) distinguish a

restricted gravity regression using sales from a single technology source across production locations and destinations from an unrestricted regression using aggregate bilateral trade. Both estimate a common tariff coefficient across country pairs. The restricted coefficient identifies $\theta/(1-\rho)$; the unrestricted coefficient is targeted by applying the regression to model-generated bilateral flows. Matching both coefficients disciplines θ and ρ. Their calibration concerns a single sector and does not provide the sector-specific values of θ_s needed in this paper.

I cannot jointly identify θ_s and ρ using the method in Arkolakis et al. (2018). The global data used in this paper do not directly report complete trilateral flows $X_{iln,s}$. The shares recovered in Section 4.1 depend on the model restrictions and therefore do not supply an independent empirical moment for estimating the conditional elasticity. Instead, I fix $\rho = 0.55$, the value calibrated by Arkolakis et al. (2018).[1] I then use the sectoral trade elasticity estimates from Caliendo and Parro (2015) to assign values to θ_s.

In this draft, I use a calibration shortcut that omits the sourcing-response term in (41) when mapping the empirical elasticities to θ_s. Let $\varepsilon_s^{\text{data}} > 0$ denote the magnitude of the sectoral empirical trade elasticity. I impose

$$\frac{\theta_s}{1-\rho} = \varepsilon_s^{\text{data}}. \tag{42}$$

Table D.1 reports the empirical trade elasticities used.

Equation (42) assigns parameter values from the empirical elasticity only after omitting the sourcing term and imposing a value for ρ. It does not match the model's full aggregate elasticity, or the model-implied gravity coefficient, to the empirical estimate. The full sourcing response is retained when solving for equilibria and optimal tariffs. A more comprehensive calibration that incorporates this response will be developed in a future draft.

4.3 Calibrating the Model with Prohibitive Multinational Production Costs

The first counterfactual is a quantitative extension of Proposition 2, in which the costs of using technology abroad are prohibitive and production is restricted to the technology's source country. I refer to the resulting economy as the *no-MP model*. It has the same sectoral Fréchet shape parameters and factor and final-consumption shares as the model with multinational production. In the no-MP model, $\gamma_{il,s} \to \infty$ for $i \neq l$, and I recalibrate technology scales $T_{il,s}$ and trade costs $\tau_{ln,s}$ such that baseline trilateral expenditure shares satisfy

$$\lambda_{iln,s}^{\text{nomp}} = \begin{cases} \lambda_{ln,s}^{\text{data}}, & i = l, \\ 0, & i \neq l. \end{cases} \tag{43}$$

Gross output and gross expenditure at baseline continue to be $Y_{n,s}^{\text{data}}$ and $X_{n,s}^{\text{data}}$.

4.4 Calibrating the Trade-Only Model

To quantify how omitting multinational production affects optimal tariffs, I calibrate a trade-only model, which I refer to as the *trade model*, to the same baseline bilateral allocation. This implements the comparison in Section 2.3.

1. Section 6 presents the sensitivity of the results to alternative values of ρ.

The trade-only model retains the preferences, input–output production structure, labor endowments, and equilibrium conditions of Section 1. Denote the country–sector productivity scale by $T_{l,s}^{\text{trade}} > 0$ and iceberg trade costs by $\tau_{ln,s}^{\text{trade}}$. Let $\theta_s^{\text{trade}} > 0$ denote the sectoral shape parameter. Productivity draws are independent across production countries and varieties, with distribution

$$\Pr\left[A_{l,s}^{\text{trade}}(\omega) \leq a\right] = \exp\left[-\left(\frac{a}{T_{l,s}^{\text{trade}}}\right)^{-\theta_s^{\text{trade}}}\right], \qquad a > 0. \tag{44}$$

This is the multisector Ricardian trade structure in Caliendo and Parro (2015). The model with multinational production in Section 1 nests this benchmark when technology-source information has no independent effect on aggregate bilateral allocation. The benchmark thus represents observed bilateral trade without separately tracking the source of the technology used to produce it. Appendix J provides further details on the link between the multinational-production and trade-only models.

Using exact-hat algebra, I jointly calibrate $T_{l,s}^{\text{trade}}$ and $\tau_{ln,s}^{\text{trade}}$ to reproduce the bilateral shares $\lambda_{ln,s}^{\text{data}} = \sum_i \lambda_{iln,s}^{\text{data}}$, gross output $Y_{n,s}^{\text{data}}$, and gross expenditure $X_{n,s}^{\text{data}}$ in the baseline. Labor and intermediate-input cost shares and final-demand shares are also identical across models. Thus, the two models start from the same baseline flows and cost and final-demand shares. They differ in whether ownership-production information disciplines how those flows respond to a tariff.

Because the trade-only model satisfies the multiplicative gravity structure, its Fréchet shape parameter directly corresponds to the empirical trade elasticity values:

$$\theta_s^{\text{trade}} = \varepsilon_s^{\text{data}}. \tag{45}$$

5 Results

The quantitative analysis studies how multinational production changes the level and structure of optimal unilateral tariffs. Throughout, Home, indexed by h, is the United States; it chooses its import tariffs while all foreign tariffs remain at zero. I refer to the full quantitative model as the *MP model* and compare it with two counterfactual economies corresponding to the analytical exercises in Section 2.

The first counterfactual shuts down multinational production by taking $\gamma_{il,s} \to \infty$ for $i \neq l$. I adjust technology scales and trade costs so that the resulting economy, the *no-MP model*, reproduces the MP model's baseline bilateral expenditure shares, gross output, and gross expenditure. Section 4.3 describes the calibration. Starting from the same bilateral trade flows, I then study how the two economies respond to a change in tariffs. This is the quantitative counterpart of the comparison in Proposition 2 and isolates the role of production-location choice.

The second counterfactual asks how accounting for multinational production in the observed allocation changes the gains from tariffs. I compare the MP model with the *trade model* calibrated in Section 4.4. Both models rationalize the same baseline bilateral allocation, but the MP model also uses ownership-production data to infer the source-specific advantages behind it. The trade model uses $\theta_s^{\text{trade}} = \theta_s/(1-\rho)$, so this comparison asks whether the information revealed by multinational production changes the response to tariffs under the current calibration. This comparison corresponds to Proposition 3.

I first study optimal uniform tariffs to understand the level of protection and relate it to the intuition in Section 2. I then examine country–sector tariffs to understand how multinational production changes tariff differentiation across origins and sectors. Finally, I measure the welfare value of this differentiation.

5.1 Optimal Uniform Tariffs

I begin by restricting the United States to a single tariff applied uniformly to every foreign origin and sector. Figures 1 and 2 trace welfare relative to setting no tariff, $\hat{U}_h \equiv U_h'/U_h$, over tariffs from 0% to 99% on a one-percentage-point grid.

Shutting down multinational production. Figure 1 compares welfare in the MP and no-MP models. The optimal tariff is 48.14% in the no-MP model and 27.21% in the MP model, with corresponding welfare gains of 0.894% and 0.497%. Both welfare curves are hump-shaped: the terms-of-trade gain dominates at low tariffs, whereas distortions to sourcing and production eventually dominate. Shutting down multinational production raises both the optimal tariff and the attainable welfare gain from protection.

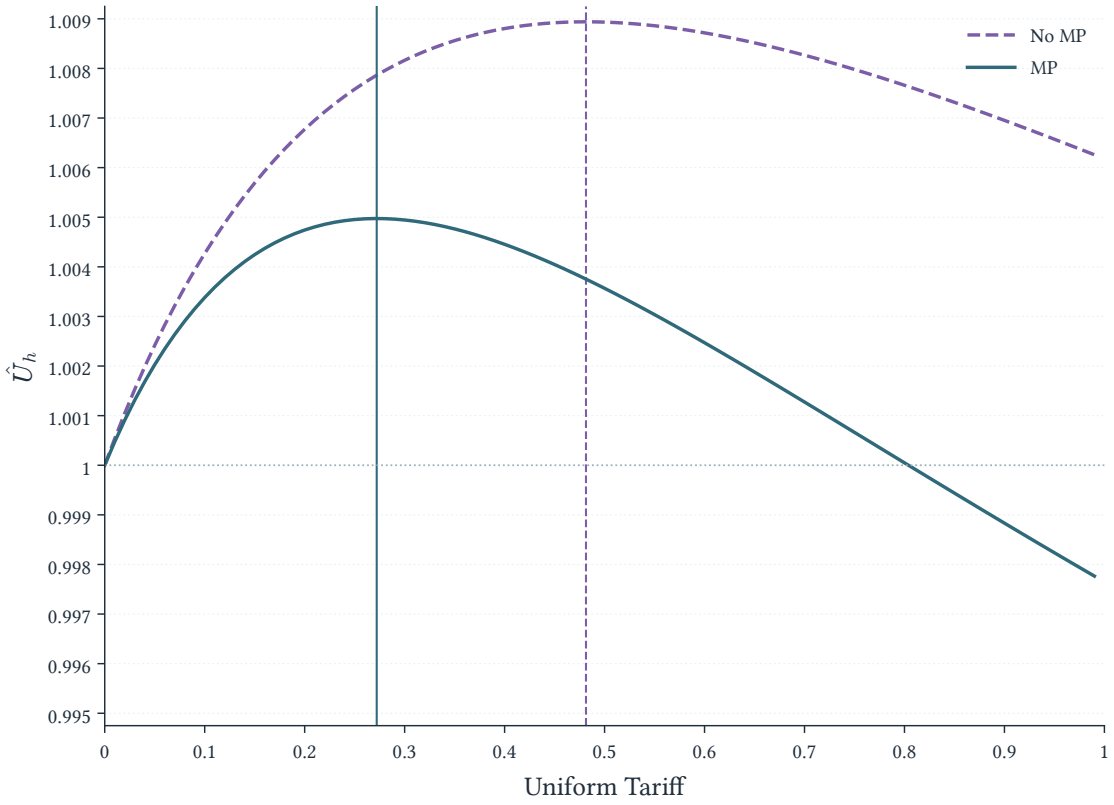


Figure 1: U.S. welfare under uniform tariffs: comparing the MP and no-MP models

Notes: The figure plots U.S. welfare relative to the zero-tariff baseline against the common U.S. tariff on all foreign origin–sector pairs. Each point is a general-equilibrium solution; vertical lines mark the optimal uniform tariffs. Foreign tariffs remain zero.

The direction of this result is consistent with Proposition 2. When multinational production is prohibited, a change in relative costs can shift expenditure between technologies but cannot change where a given technology produces. This lowers the export-supply elasticity relative to the economy in which multinational production is allowed. When production is allowed to relocate, an additional margin of adjustment opens, raising the export-supply elasticity. Foreign labor demand can adjust more through relocation, weakening the terms-of-trade incentive to raise the tariff. Equation (A.12) in Appendix A gives the corresponding sourcing elasticities with many countries and sectors. They retain the production-location mechanism from Section 2, while wages and sectoral input prices transmit cost changes through the additional trade and input–output linkages.

Accounting for observed multinational production. Figure 2 compares the MP and trade models. The optimal uniform tariff is 22.01% in the trade model, yielding a welfare gain of 0.412%, compared with an optimal uniform tariff of 27.21% and a welfare gain of 0.497% in the MP model. Thus, accounting for multinational production raises the optimal tariff by 5.20 percentage points under the current calibration.

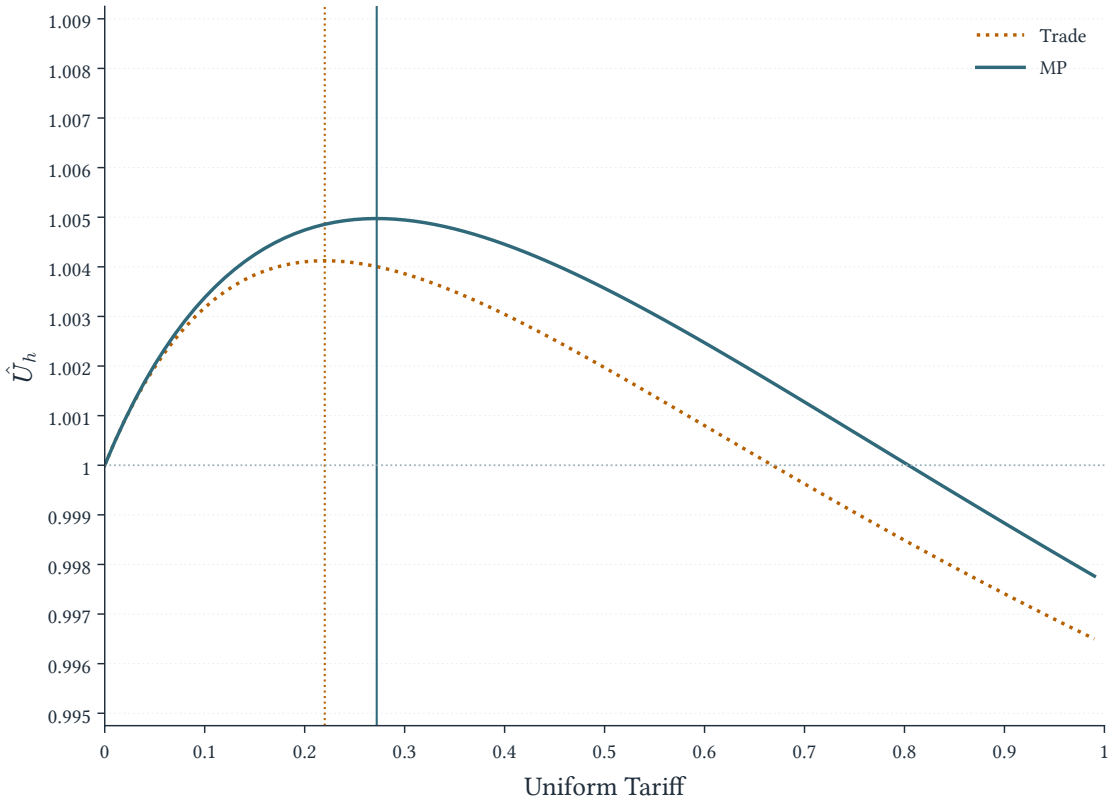


Figure 2: U.S. welfare under uniform tariffs: comparing the MP and trade models

Notes: The figure plots U.S. welfare relative to the zero-tariff baseline against the common U.S. tariff on all foreign origin–sector pairs. Each point is a general-equilibrium solution; vertical lines mark the optimal uniform tariffs. Foreign tariffs remain zero.

Proposition 3 provides the intuition for the opposite result. The same bilateral shares can combine technology sources with similar advantages across production locations or sources with sharply different advantages across locations. In the latter case, a given change in relative costs shifts less production than the trade model infers from bilateral shares. The bilateral flows alone conceal the extent to which technologies specialize in particular production locations. Accounting for this specialization implies that larger cost changes are needed to induce firms to relocate production. The export-supply elasticity is therefore lower, strengthening the terms-of-trade incentive to raise the tariff. Figures 3 and 4 connect this intuition to the production-location choices of each technology source observed in the baseline and the resulting equilibrium response to tariffs.

The analytical comparison in Section 2 relates the export-supply elasticity to the dispersion in production propensities across technology sources. Figure 3 gives the corresponding comparison in the quantitative economy. It summarizes each technology source's propensity to serve foreign countries through production in the United States. For each technology source i, it reports the fraction of its sales to foreign

destinations that is produced in the United States:

$$\frac{\lambda_{ihf}}{\sum_l \lambda_{ilf}} \equiv \frac{\sum_{n \neq h,s} X_{ihn,s}}{\sum_{l,n \neq h,s} X_{iln,s}}. \tag{46}$$

Here f pools all foreign destinations, and expenditure is aggregated across sectors. U.S. technology produces 32.32% of its sales to foreign markets in the United States, whereas the corresponding fraction is at most 1.71% for the other technology sources. The dispersion across sources in Figure 3 reveals each technology source's advantages across production locations that the aggregate bilateral share λ_{hf} conceals. In the model with multiple countries and sectors, equation (A.12) in Appendix A shows how differences in production propensities across technology sources affect sourcing responses to relative costs. These responses, together with the adjustment of wages and input prices, determine export supply.

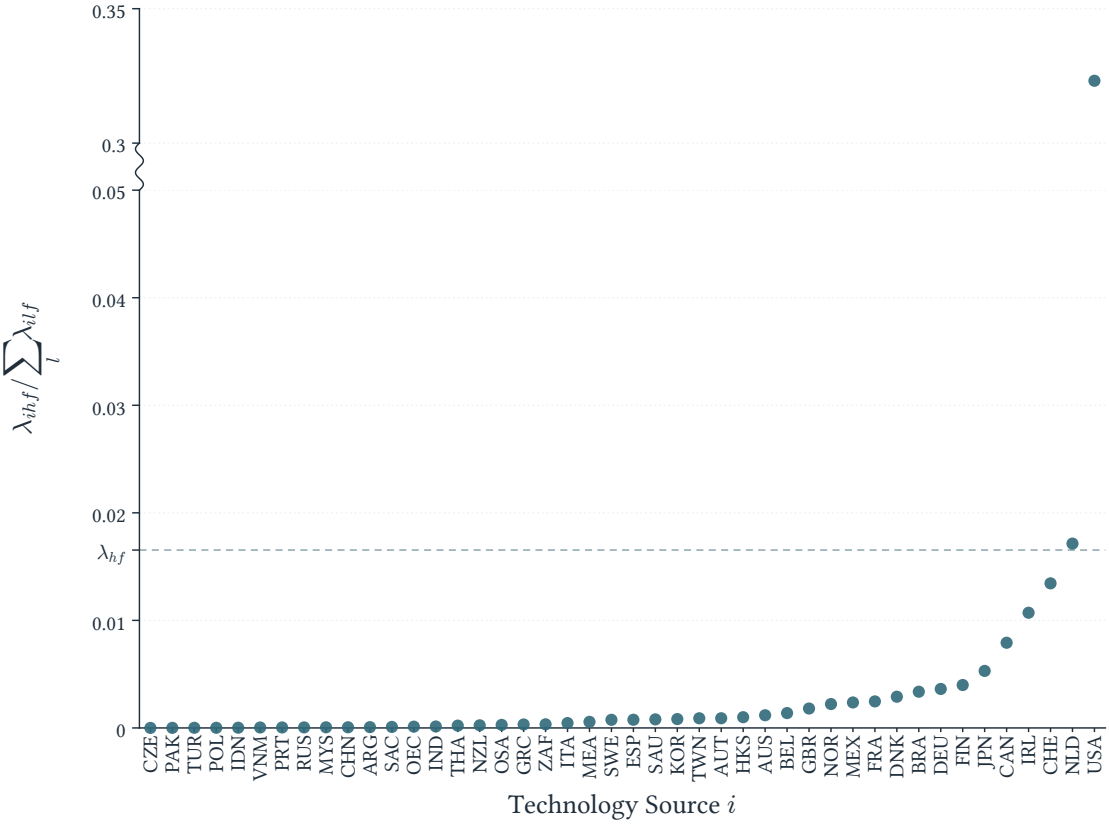


Figure 3: Propensity to produce in the United States by technology source i

Notes: Each marker is the fraction defined by (46) for one technology-source country at the baseline equilibrium. ROW is omitted from the figure but included in the calculations. The dashed line is the aggregate U.S. production share in foreign expenditure, $\lambda_{hf} = 0.0165$.

To examine how these production-location advantages affect the response to a tariff, consider foreign expenditure on U.S.-produced goods. Define the expenditure share and its change from the baseline as

$$\lambda'_{hf} \equiv \frac{\sum_{n\neq h,s} \lambda'_{hn,s} X'_{n,s}}{\sum_{n\neq h,s} X'_{n,s}}, \qquad \hat{\lambda}_{hf} \equiv \frac{\lambda'_{hf}}{\lambda_{hf}}. \tag{47}$$

Figure 4 plots $\hat{\lambda}_{hf}$ against the uniform U.S. tariff in the MP and trade models. The tariff reduces demand for foreign production and puts downward pressure on foreign wages relative to the U.S. wage. This makes U.S.-produced goods more expensive relative to foreign alternatives, so their share in foreign expenditure falls. The decline is smaller in the MP model, in which the observed concentration of production by technology source implies stronger advantages in particular locations. This smaller equilibrium response is consistent with the lower export-supply elasticity at the common allocation in Proposition 3 and the higher optimal tariff. The curves include the adjustment of wages, prices, and expenditure; they do not directly measure the partial elasticity in (28).

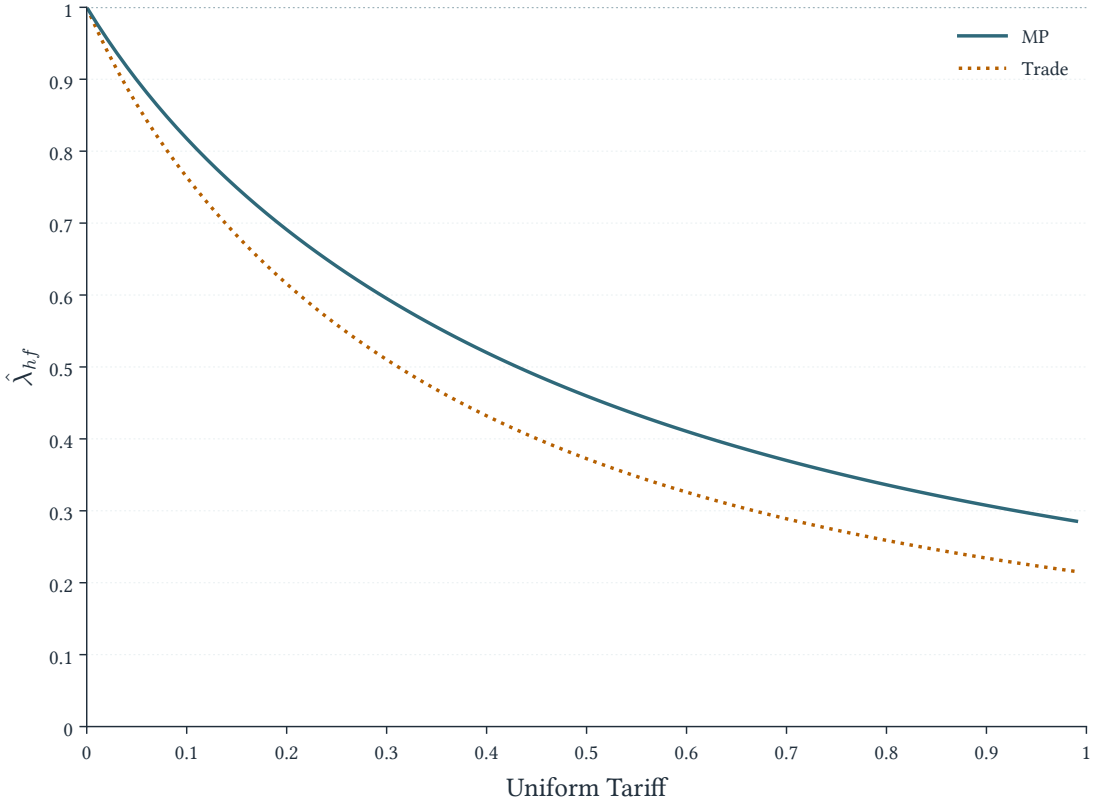


Figure 4: Change in foreign expenditure on U.S.-produced goods in response to a uniform U.S. tariff

Notes: The vertical axis is $\hat{\lambda}_{hf}$ in (47); the horizontal axis is the common U.S. tariff on all foreign origin–sector pairs. Each point is a general-equilibrium solution. Foreign tariffs remain zero.

5.2 The Structure of Optimal Country-Sector Tariffs

I now allow tariffs to vary across foreign origins and sectors. Figure C.3 in Appendix C shows that the optimal origin–sector tariffs vary substantially across both countries and sectors in the MP model. Here I focus on the counterfactual comparisons to examine the role of multinational production in this variation. I compare tariffs for the same origin–sector pairs to distinguish changes in the overall level of protection from changes in which origins and sectors receive higher tariffs. I then examine their distributions across sectors and origins to see how the pattern of differentiation changes across models.

Figures 5 and 7 exclude mining and petroleum from the exporter–sector tariff comparisons. The paper does not establish that the optimal tariff vector is unique. To assess whether the numerical solution depends on the initial tariff guess, I solve the tariff problem from five different initial schedules. As shown in Section 6.2, these searches attain nearly identical welfare levels but yield different tariffs for some exporter–sector pairs, with the largest differences occurring in petroleum in the MP model and mining in the trade model. The affected pairs have small baseline import expenditure shares, and both sectors have high empirical trade elasticity inputs. Strong substitution as tariffs rise can leave a small remaining import base, making welfare relatively insensitive to further changes in these tariffs. This sensitivity of individual rates motivates excluding the two sectors from these pairwise comparisons.

Shutting down multinational production. Figure 5 compares the computed optimal tariffs for each origin–sector pair in the MP and no-MP models. As in the uniform-tariff exercise, allowing multinational production generally lowers protection: 99.02% of the plotted tariffs lie below the 45-degree line. The expenditure-weighted line of best fit has a slope of 0.731, and the expenditure-weighted Spearman rank correlation is 0.946, indicating that the relative pattern of higher and lower tariffs across origin–sector pairs changes little. Figure C.4 in Appendix C retains mining and petroleum and has a slope of 0.712 and a rank correlation of 0.926, so including these sectors preserves the broad comparison.

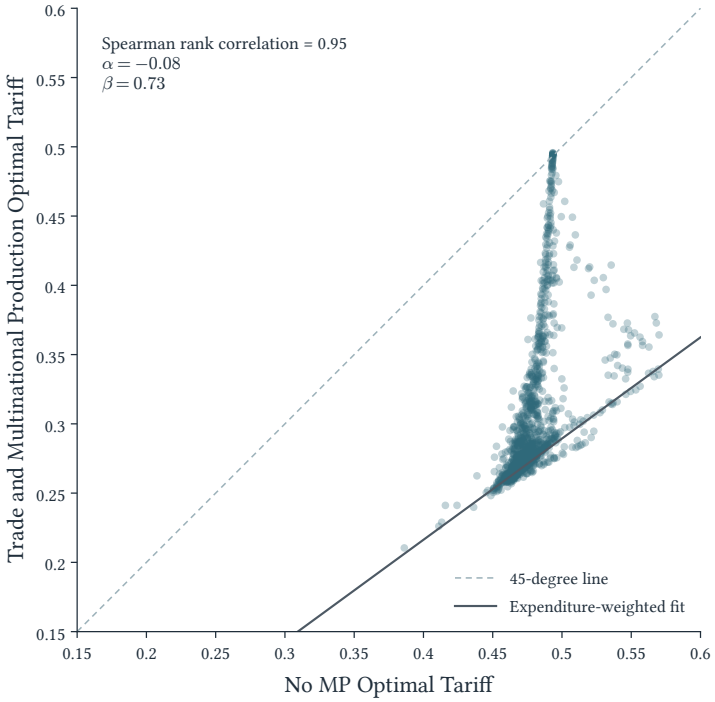


Figure 5: Optimal U.S. country–sector tariffs: comparing the MP and no-MP models

Notes: Each marker represents one of 1,333 foreign-origin–sector pairs. ROW is included in the tariff optimization but omitted from the figure. The Spearman correlation measures agreement in the ranking of these pairs across models; a value of one indicates identical rankings. The fitted line and Spearman correlation use baseline bilateral expenditure weights, $\lambda_{lh,s} X_{h,s}$. Mining and petroleum are excluded because their tariffs are sensitive to the initial tariff vector, making it difficult to obtain a unique numerical schedule across starts, as discussed in Section 6. Both sectors remain in the optimization. In the figure, α is the intercept and β is the slope of the fitted line.

Figure 6 compares the distributions of these schedules. The MP distributions lie at substantially lower tariff levels, while sectors with relatively high or low tariffs generally retain their positions across the two models. The MP model also exhibits greater dispersion within sectors and origins. The median within-sector span from the 10th to the 90th percentile is 7.14 percentage points in the MP model, compared with 2.28 in the no-MP model. The corresponding median within-origin spans are 12.52 and 1.85 percentage points. Allowing relocation therefore changes the extent of differentiation within each group as well as the overall level of tariffs. As discussed in Section 3.2, relocation affects the exporter-specific wage response and the way a sectoral cost change propagates through the production network. These effects differ across origins and sectors because their exposure to production-location choices and input–output linkages differs.

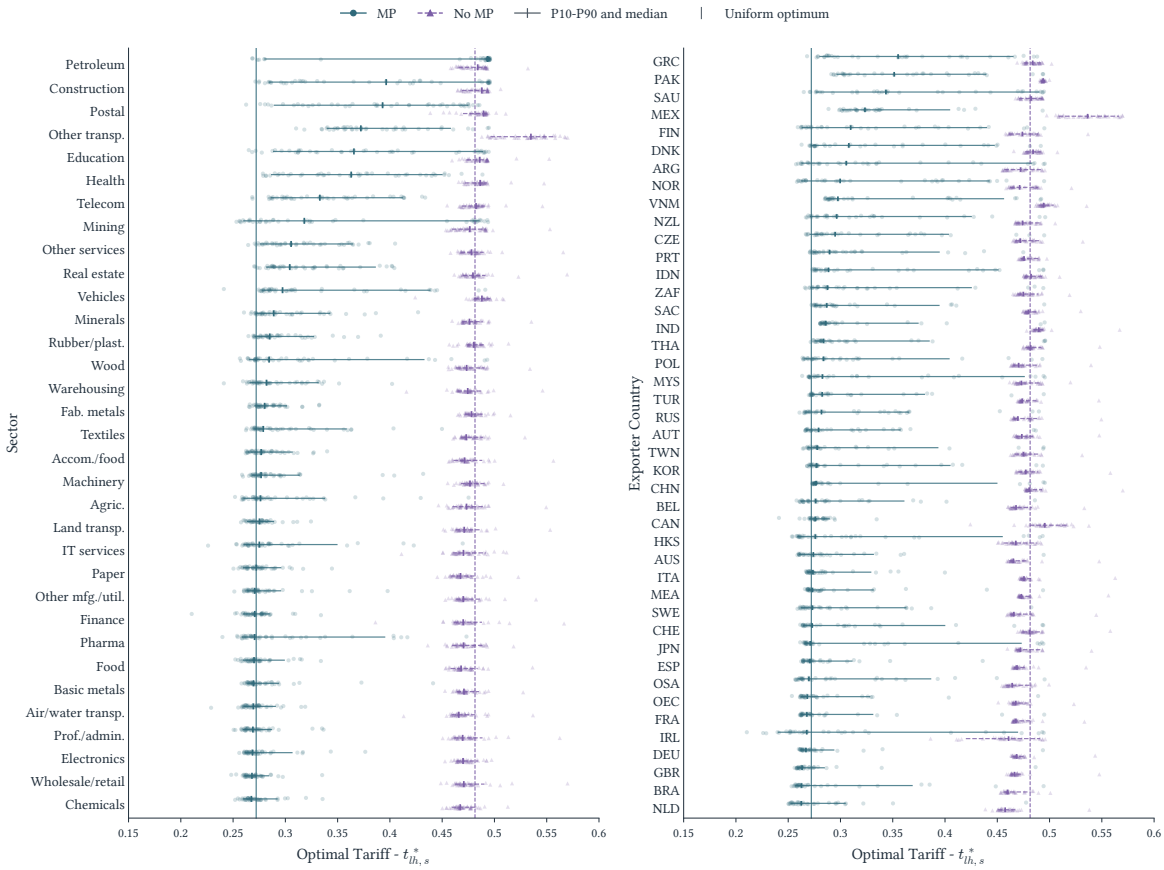


Figure 6: Country–sector tariff distributions in the MP and no-MP models

Notes: The left panel groups tariffs by sector and the right panel by foreign origin. Dots are individual origin–sector tariffs, horizontal segments span the 10th–90th percentiles, and vertical ticks mark medians. Dotted vertical lines are the models' optimal uniform tariffs. ROW is included in the equilibrium and tariff optimization but omitted from the figure.

Accounting for observed multinational production. Figure 7 compares the MP and trade models. The dominant feature is now an upward shift: MP tariffs exceed trade-model tariffs for 98.20% of the plotted pairs. The expenditure-weighted line of best fit has a slope of 1.292, indicating that the fitted increase is larger at higher trade-model tariffs in this restricted sample. The expenditure-weighted Spearman rank correlation is 0.943, indicating that the relative ranking remains similar. Accounting for technology-source sorting therefore changes the level of protection much more than its ranking in this comparison, paralleling the higher optimal uniform tariff. Figure C.5 in Appendix C includes mining and petroleum. The rank correlation remains high at 0.935, but the slope falls to 0.683. Thus, the similarity in rankings extends to the comparison with all sectors, while the slope interpretation depends on the exclusion of mining and petroleum.

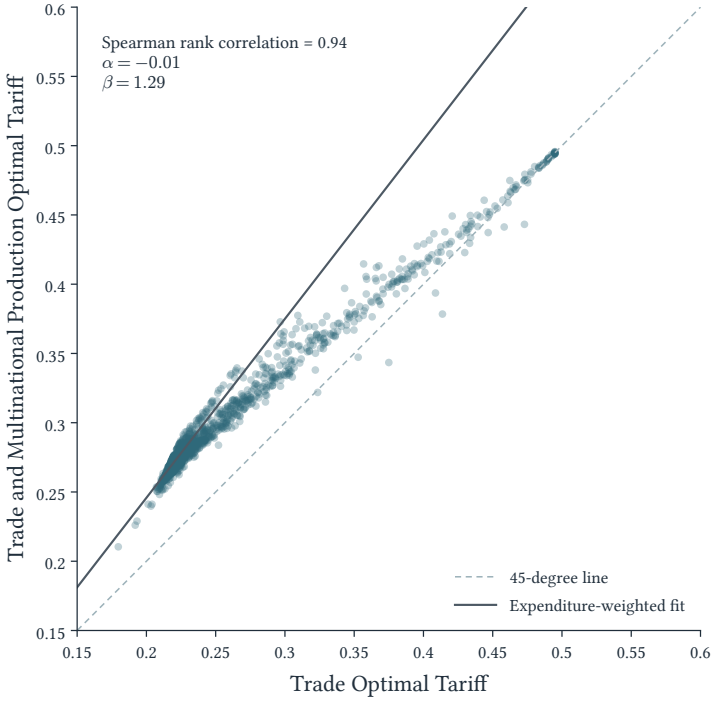


Figure 7: Optimal U.S. country–sector tariffs: comparing the MP and trade models

Notes: Each marker represents one of 1,333 foreign-origin–sector pairs. ROW is included in the tariff optimization but omitted from the figure. The Spearman correlation measures agreement in the ranking of these pairs across models; a value of one indicates identical rankings. The fitted line and Spearman correlation use baseline bilateral expenditure weights, $\lambda_{lh,s} X_{h,s}$. Mining and petroleum are excluded because their tariffs are sensitive to the initial tariff vector, making it difficult to obtain a unique numerical schedule across starts, as discussed in Section 6. Both sectors remain in the optimization. In the figure, α is the intercept and β is the slope of the fitted line.

Figure 8 shows that the MP and trade models also have similar patterns of differentiation across sectors and origins. Most sector medians shift upward with MP, consistent with the level shift in Figure 7. The common bilateral allocation and production network largely preserve which origins and sectors receive higher tariffs, while technology-source sorting changes the strength of the terms-of-trade incentive. Within either model, sectoral and origin medians conceal substantial variation across individual origin–sector pairs, so neither dimension alone organizes the schedule.

The analytical comparison provides one way to interpret the differences in dispersion. At common bilateral shares, the trade model has a higher export-supply elasticity than the no-MP model: its shape parameter is $\theta_s/(1-\rho)$, compared with θ_s in the no-MP model. These sourcing sensitivities enter the equilibrium wage responses described in Section 3.2. The MP and trade models share the same conditional cost sensitivity, although technology-source sorting makes their full sourcing responses differ. This common sensitivity may help explain why their tariff distributions are more similar. The median within-origin 10th–90th percentile spans are 12.52 percentage points in the MP model and 15.96 in the trade model, compared with 1.85 in the no-MP model. The sourcing elasticities help interpret these distributions, but do not by themselves establish a ranking of tariff dispersion across models.

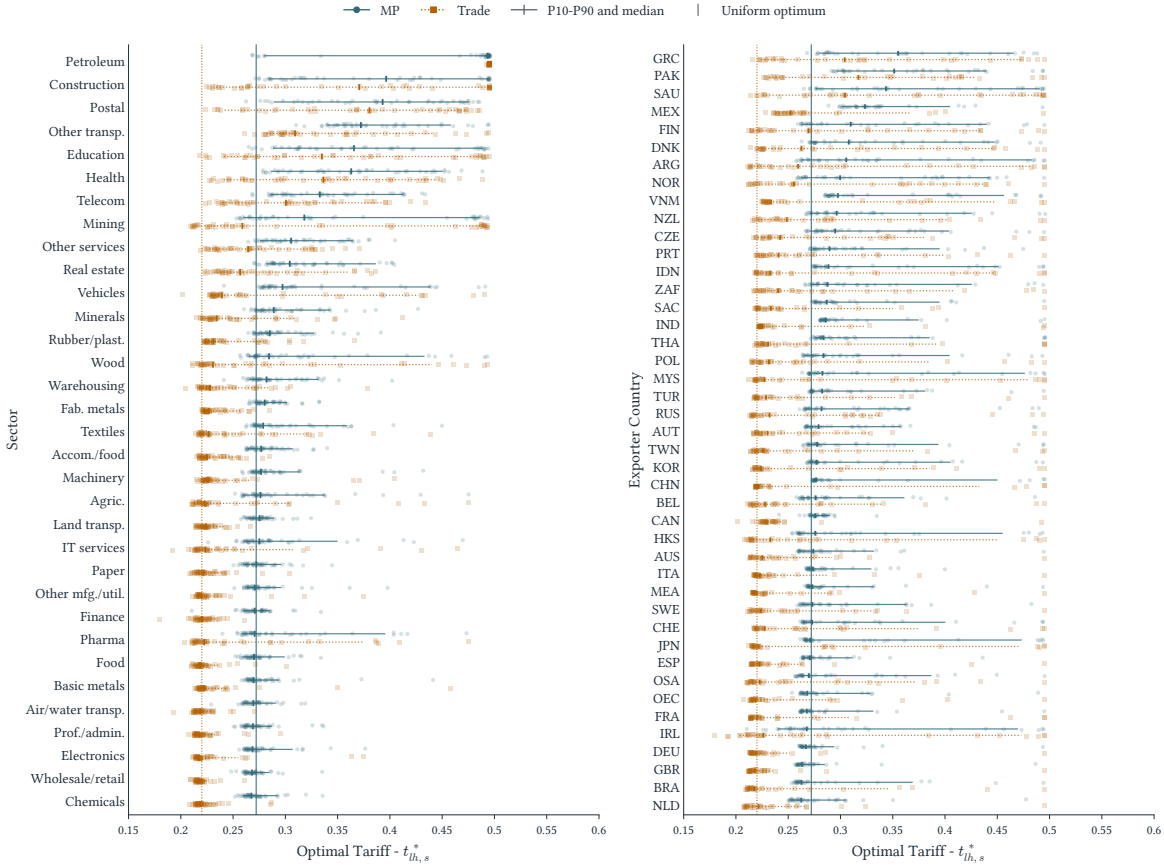


Figure 8: Country–sector tariff distributions in the MP and trade models

Notes: The left panel groups tariffs by sector and the right panel by foreign origin. Dots are individual origin–sector tariffs, horizontal segments span the 10th–90th percentiles, and vertical ticks mark medians. Dotted vertical lines are the models' optimal uniform tariffs. ROW is included in the equilibrium and tariff optimization but omitted from the figure.

5.3 The Welfare Value of Tariff Granularity

The dispersion in exporter–sector tariffs in the model with multinational production need not imply large welfare gains from differentiation. I reoptimize tariffs under four instrument sets: a common tariff, one tariff per sector, one tariff per foreign origin, and one tariff per origin–sector pair. Comparing the resulting welfare gains measures the value of exploiting the sectoral and exporter-specific responses described in Section 3.2. Figures C.1–C.3 in Appendix C show the optimal schedules under the three forms of differentiation.

Table 2: Welfare changes under optimal tariffs (%)

Tariff Granularity	welfare gain (%)
scalar	0.497
sectors	0.498
country	0.499
sector-country	0.500

Note: Entries report percentage welfare changes, $100(\hat{U}_h - 1)$, for USA under the optimal unilateral tariff at each level of tariff granularity, using 2020 baseline data in the model with multinational production (MP).

Table 2 shows that a uniform tariff captures more than 99% of the attainable welfare gain. Restricting policy to a single uniform tariff raises household real consumption by 0.497% relative to free trade, compared with 0.500% under the fully differentiated schedule across all 1,452 origin–sector pairs. Differentiating import duties by origin and sector therefore delivers an incremental welfare gain of only 0.003 percentage points. From an economic and policy standpoint, this small difference indicates that the terms-of-trade benefits accessible through detailed country–sector discrimination are almost entirely captured by a uniform import tariff, leaving little welfare return to the informational and administrative complexity of implementing 1,452 distinct tariff lines.

6 Robustness

6.1 Sensitivity of Results to the Value of ρ

The parameter ρ governs the correlation of a technology source's productivity across production locations. Like θ_s, it is a key parameter determining trade elasticities and hence optimal tariffs for a given baseline equilibrium. Furthermore, as described in Section 4, its chosen value determines the trilateral trade flows recovered from the trade and ownership data. Removing trade imbalances from these data is also model-based and depends on ρ. I examine the sensitivity of the results in Section 5 to the calibrated value of $\rho = 0.55$ by considering $\rho \in \{0, 0.30, 0.60, 0.90\}$.

For each value of ρ, I follow the calibration steps in Section 4 for both the MP model and the trade-only model. Holding $\theta_s/(1-\rho) = \varepsilon_s^{\text{data}}$ fixed at the empirical sectoral trade elasticity keeps the conditional sensitivity of location choices within a technology source constant across specifications. Consequently, varying ρ from zero to 0.90 mechanically scales the structural shape parameter $\theta_s = (1-\rho)\varepsilon_s^{\text{data}}$ from the full empirical elasticity down to one tenth of that value. The exercise therefore assesses joint sensitivity to cross-location productivity correlation and the dispersion of productivity across varieties within a technology source, rather than an isolated change in location correlation. I compute welfare gains relative to the zero-tariff baseline in each calibration.

Table 3: Optimal Uniform Tariffs Across Values of ρ

ρ	Optimal uniform tariff (%)		Welfare gain (%)	
	MP	Trade	MP	Trade
0.00	22.03	22.03	0.41	0.41
0.30	24.39	22.02	0.45	0.41
0.55 (baseline)	27.21	22.01	0.50	0.41
0.60	27.89	22.01	0.51	0.41
0.90	33.46	22.00	0.63	0.42

Note: Results use 2020 data for USA. Welfare gains are $100(\hat{U}_h - 1)$ relative to the zero-tariff baseline in each calibration. Foreign tariffs remain zero.

Table 3 shows that the optimal uniform tariff in the MP model rises from 22.03% at $\rho = 0$ to 33.46% at $\rho = 0.90$, while the trade model's tariff remains close to 22%. A similar pattern appears in the welfare gains: the MP model's gain rises from 0.41% to 0.63%, compared with an increase from 0.41% to 0.42% in the trade model. At $\rho = 0$, the models have the same optimal uniform tariff and welfare gain. As ρ rises, so does the gap between the models' optimal tariffs. The difference is 5.20 percentage points under the preferred calibration and 11.46 percentage points at $\rho = 0.90$.

The role of ρ can be understood by considering what technology ownership reveals about production possibilities across locations. When $\rho = 0$, a technology's productivity draws in different locations are independent. The best technology available in each location can then be summarized by a location-specific productivity distribution. Differences in productivity and multinational-production costs across sources are absorbed into these effective productivity scales, so bilateral trade responses do not require separately tracking technology ownership, as shown in Appendix J. Multinational production can occur, but its ownership composition adds no information needed to predict these aggregate responses.

For positive ρ, a technology's productivity across locations is correlated, so its production possibilities in one country are linked to those in others. Observing where each technology source produces then provides information about its relative advantages across locations that bilateral trade flows alone conceal. A source that concentrates production in one location despite available alternatives reveals a strong advantage there and adjusts less to a given change in relative costs. As discussed in Section 2, accounting for these source-specific advantages lowers export-supply responsiveness relative to the trade model and strengthens the terms-of-trade incentive for protection. Under the mapping in (42), however, higher ρ also lowers θ_s, increasing productivity dispersion across varieties within each technology source. The observed expansion in the optimal-tariff gap therefore reflects the combined effect of increasing cross-location productivity correlation and increasing productivity dispersion, together with the associated recalibration of the baseline; it cannot be attributed to correlation alone.

Table 4: Exporter–Sector Tariff Comparisons Across Values of ρ

ρ	Slope of best fit	Spearman rank correlation
0.00	1.00	1.00
0.30	1.13	0.98
0.55 (baseline)	1.29	0.94
0.60	1.33	0.93
0.90	1.66	0.84

Note: Results use 2020 data for USA. The slope is from a weighted least-squares regression of optimal MP tariffs on optimal Trade tariffs, including an intercept. The fit and Spearman correlation use bilateral expenditure weights $\lambda_{lh,s}X_{h,s}$ from the baseline calibration for each value of ρ. ROW, mining, and petroleum are excluded from both statistics but remain in the equilibrium and tariff optimization.

Table 4 shows how the comparison changes when tariffs can vary across exporters and sectors. The slope of the expenditure-weighted line of best fit rises from 1.00 at $\rho = 0$ to 1.66 at $\rho = 0.90$, compared with 1.29 under the preferred calibration. Thus, the increase in protection associated with accounting for multinational production becomes larger as ρ rises. The weighted Spearman correlation falls from 1.00 to 0.84, compared with 0.94 at $\rho = 0.55$. Thus, along this joint change in correlation, productivity dispersion, and the calibrated baseline, optimal exporter–sector targeting also changes relative to the trade model.

6.2 Sensitivity to the Initial Tariff Vector

The paper does not establish that the equilibrium for a given tariff vector is unique or that the optimal tariff vector is unique. The numerical optimization algorithm described in Section 3 begins from an initial tariff guess and updates the vector from that starting point. Different guesses may lead to different local optima, with different tariff vectors and attained welfare levels. For uniform tariffs, Figures 1 and 2 show clearly hump-shaped welfare curves with a single maximum over the feasible tariff range. The global optimum can therefore be identified directly from these one-dimensional welfare profiles. When tariffs vary jointly across exporters and sectors, the concerns are whether the algorithm finds a local rather than a global optimum and whether the tariff vector attaining that optimum is unique. Thus, I solve the exporter–sector problem from five different initial tariff vectors. The schedule attaining the highest welfare is used for the results in Section 5.

Table 5 shows that the attained welfare level is effectively unchanged across starts. The welfare comparisons are therefore insensitive to these changes in initialization.

Table 5: Variation in Optimal Tariffs Across Starts

Model	Max. Welfare Gap	Maximum Absolute Tariff Deviation				
		Min.	5%	50%	95%	Max.
MP	7.84×10^{-7}	0.02×10^{-2}	0.04×10^{-2}	0.30×10^{-2}	1.88×10^{-2}	21.34×10^{-2}
No MP	0.05×10^{-7}	0.00×10^{-2}	0.01×10^{-2}	0.06×10^{-2}	0.52×10^{-2}	0.92×10^{-2}
Trade	11.15×10^{-7}	0.03×10^{-2}	0.04×10^{-2}	0.25×10^{-2}	2.13×10^{-2}	10.60×10^{-2}

Note: Results use 2020 baseline data for USA. The welfare gap is $\max_r \hat{U}_{h,r} - \min_r \hat{U}_{h,r}$, in original welfare units. For each exporter-sector, the maximum absolute tariff deviation is taken over all ten distinct pairs of the five starts. The remaining columns summarize these cell maxima, with equal weight per exporter-sector, in original tariff units. ROW is excluded from the summary statistics.

Most individual tariffs also change little across starts, although some differ materially. For each exporter–sector pair, the table reports the largest absolute tariff difference across runs. The 95th percentile of these maximum differences is 1.88 percentage points in the MP model, 0.52 in the no-MP model, and 2.13 in the trade model.

The largest differences occur in petroleum in the MP model and mining in the trade model. Tables C.1–C.3 in Appendix C identify the five exporter–sector pairs with the largest tariff ranges in each model. The five petroleum pairs in the MP model together account for 0.30% of baseline U.S. import expenditure, while the five mining pairs in the trade model account for 0.01%. Both sectors also have high empirical elasticity inputs, as reported in Table D.1. The small import weights of these exporter–sector pairs, together with strong substitution away from them as tariffs rise, can leave the welfare objective relatively flat: further tariff changes affect a small remaining import base. This helps explain why large differences in these tariffs coexist with negligible welfare differences. These results motivate excluding mining and petroleum from the 45-degree comparisons in Section 5, while retaining them in the tariff distributions and welfare analysis.

Figure 9 shows the cumulative distributions of absolute tariff deviations from the vector attaining the highest welfare, with equal weight on each exporter–sector–run comparison. Most differences are small, but the MP and trade models have a longer upper tail than the no-MP model.

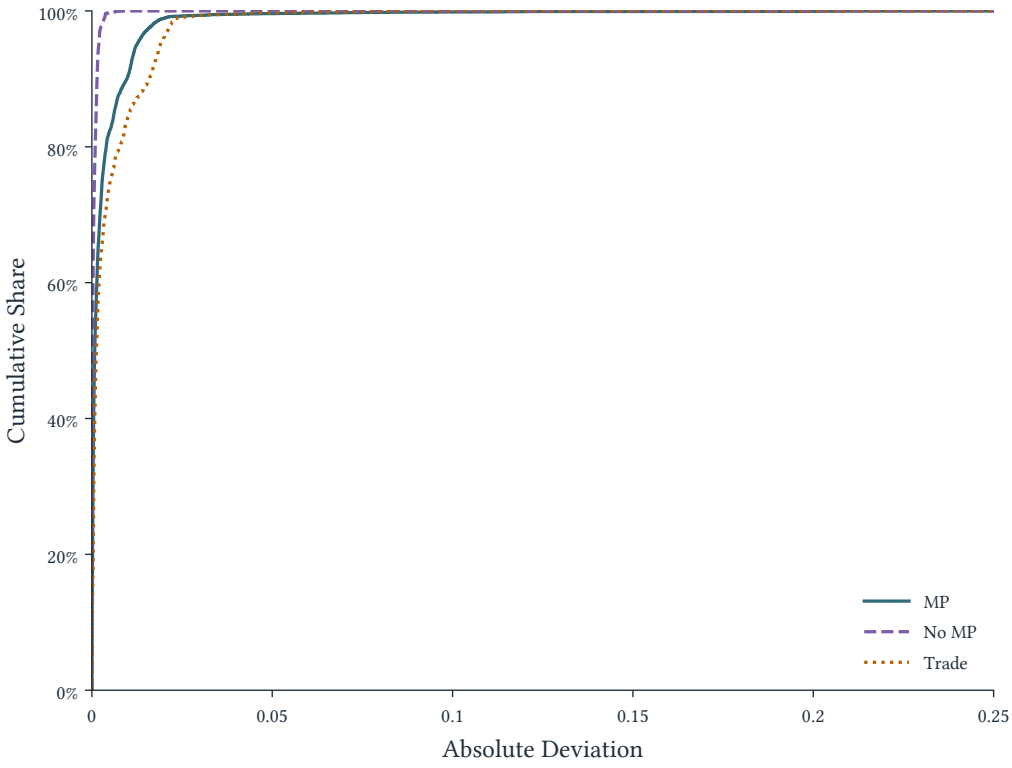


Figure 9: Sensitivity of Optimal Tariffs to the Initial Tariff Vector

Notes: Each curve pools absolute tariff differences between the alternative starting runs and the run attaining the highest welfare, with equal weight per exporter–sector–run comparison. ROW is omitted from the figure but included in the equilibrium and optimization.

The coexistence of negligible welfare differences across starts with non-trivial tariff dispersion among specific exporter–sector pairs indicates that the welfare objective is relatively flat across a multidimensional range of tariff schedules. Beyond its computational implications, this finding qualifies the economic interpretation of optimal differentiated tariffs. Because substantial variations in individual origin–sector rates produce very small differences in aggregate welfare, the exact vector of differentiated tariffs is not tightly pinned down for practical policy purposes at the numerical precision of this exercise. This reinforces the finding in Section 5.3 that fine-tuning tariffs across individual foreign origins and sectors provides negligible welfare value over a transparent uniform tariff. The multi-start results support the stability of the welfare comparisons, but do not establish global optimality or mathematical uniqueness of the tariff schedule.

Conclusion

Multinational firms account for a large share of international trade, yet their implications for optimal import tariffs remain less understood. This paper distinguishes the effect of making production abroad feasible from the effect of accounting for multinational production already embodied in observed trade. The analytical results connect both questions to the export-supply elasticity. Additional production locations create another margin of adjustment, while observed choices reveal source-specific location advantages that can make adjustment less responsive. Bilateral trade shares alone conceal these advantages, so models that reproduce the same trade flows can imply different optimal policies.

The quantitative model combines multinational production with multiple countries, sectors, and input–output linkages. An analytical welfare gradient makes the differentiated tariff problem tractable while retaining the full general-equilibrium response of wages, prices, and sourcing. Under the current calibration, making multinational production feasible lowers the optimal uniform U.S. tariff by 20.9 percentage points, while accounting for observed multinational production raises it by 5.2 percentage points relative to a trade-only model. The exporter–sector results primarily show changes in tariff levels, with similar rankings across the comparisons. Despite dispersion across individual tariffs, a uniform tariff captures almost all the welfare gain: differentiation adds only 0.003 percentage points. The small gain emerges after allowing tariffs to exploit both production-network effects and exporter-specific wage responses.

The framework extends naturally to several policy settings. First, this paper holds foreign trade policies fixed to isolate the unilateral terms-of-trade motive. Allowing trading partners to retaliate would alter the welfare consequences of both uniform and differentiated tariffs, making the characterization of multi-country tariff games a natural next step. Second, the policy instruments analyzed here are restricted to import tariffs. Incorporating export subsidies can help assess whether additional welfare gains can be attained through a dual policy approach that targets production more directly. The central empirical lesson remains: bilateral trade flows alone are insufficient to predict the response to trade policy. Whether firms have the capacity to relocate production, and how strongly observed sorting ties them to specific locations, governs whether adjustment occurs through prices or quantities, determining the incidence of protection.

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A Quantitative Model

This appendix provides supporting derivations for the model in Section 1. It derives sectoral prices and expenditure shares, characterizes trade elasticities, and obtains the equations used in the equilibrium solver. It also supports the welfare objective and optimal tariff problem stated in Section 3. The equivalent cross-nested CES representation is presented separately in Appendix K.

A.1 Derivation of Price Indices and Expenditure Shares

Fix a destination n and sector s. For the derivations in this appendix, define the normalized tariff-inclusive cost:

$$c_{iln,s} \equiv \tilde{\gamma}_{il,s}\tilde{\tau}_{ln,s}(1 + t_{ln,s}), \tag{A.1}$$

and the cost-adjusted technology terms

$$D_{iln,s} \equiv c_{iln,s}^{-\frac{\theta_s}{1-\rho}}, \qquad D_{in,s} \equiv \sum_l D_{iln,s}, \qquad D_{n,s} \equiv \sum_i D_{in,s}^{1-\rho}. \tag{A.2}$$

We use the max-stability and sourcing properties of the multivariate Fréchet distribution in Lind and Ramondo (2023). Applying (2) to cost-adjusted productivity and using independence across technology sources i gives the joint distribution across all source–location pairs:

$$\Pr\left[\frac{A_{il,s}(\omega)}{T_{il,s}c_{iln,s}} \le a_{il,s} \text{ for all } i,l\right] = \exp\left\{-\sum_i\left(\sum_l D_{iln,s}a_{il,s}^{-\frac{\theta_s}{1-\rho}}\right)^{1-\rho}\right\}.$$

The exponent is homogeneous of degree one in the terms $a_{il,s}^{-\theta_s}$. Setting every threshold equal to a therefore gives the distribution of the largest cost-adjusted productivity:

$$\Pr\left[\max_{i,l}\frac{A_{il,s}(\omega)}{T_{il,s}c_{iln,s}} \le a\right] = \exp\left[-a^{-\theta_s}\sum_i D_{in,s}^{1-\rho}\right] = \exp(-D_{n,s}a^{-\theta_s}). \tag{A.3}$$

By (1) and (A.1), the price is the reciprocal of this maximum. Thus

$$\Pr[p_{n,s}(\omega) \le p] = 1 - \exp(-D_{n,s}p^{\theta_s}), \qquad p > 0.$$

The corresponding Fréchet moment, together with the law of large numbers across independent varieties, gives

$$\int_0^1 p_{n,s}(\omega)^{1-\sigma_s}\,d\omega = D_{n,s}^{\frac{\sigma_s-1}{\theta_s}}\Gamma\left(1 - \frac{\sigma_s - 1}{\theta_s}\right),$$

provided $\theta_s > \sigma_s - 1$. Raising this to the power $1/(1-\sigma_s)$ yields the sectoral price index in (4).

To obtain the sourcing probability, differentiate the joint distribution with respect to the threshold for pair (i,l) and set all thresholds equal to a. This gives the density of the maximum when that pair supplies the

good:

$$\Pr\left[(i,l) \text{ supplies } \omega \text{ in } n,\ \max_{j,m} \frac{A_{jm,s}(\omega)}{T_{jm,s}c_{jmn,s}} \in da\right] = \theta_s a^{-\theta_s-1} D_{iln,s} D_{in,s}^{-\rho} e^{-D_{n,s}a^{-\theta_s}}\, da.$$

Integrating from zero to a, and using that the maximum is the reciprocal of price, gives

$$\begin{aligned}&\Pr[(i,l) \text{ supplies } \omega \text{ in } n,\ p_{n,s}(\omega) \geq 1/a] \\ &\quad = \frac{D_{iln,s} D_{in,s}^{-\rho}}{D_{n,s}} \Pr[p_{n,s}(\omega) \geq 1/a].\end{aligned} \tag{A.4}$$

The factor does not depend on a, so the supplying pair and its price are independent. Summing that factor over production locations l gives the probability $D_{in,s}^{1-\rho}/D_{n,s}$ that technology from i supplies the good. Conditional on source i, the probability of production in l is $D_{iln,s}/D_{in,s}$. Their product is the probability that pair (i,l) supplies the good.

CES demand makes expenditure on a variety proportional to $p_{n,s}(\omega)^{1-\sigma_s}$. Independence of price and supplier in (A.4), together with the law of large numbers across independent varieties, therefore converts that sourcing probability into the expenditure share:

$$\lambda_{iln,s} = \frac{\mathbb{E}[p_{n,s}(\omega)^{1-\sigma_s} \mathbf{1}\{(i,l) \text{ supplies } \omega \text{ in } n\}]}{\mathbb{E}[p_{n,s}(\omega)^{1-\sigma_s}]} = \frac{D_{iln,s}}{D_{in,s}} \frac{D_{in,s}^{1-\rho}}{D_{n,s}}. \tag{A.5}$$

This yields (7). Within source i, $D_{in,s}^{-\rho}/D_{n,s}$ cancels from $\lambda_{iln,s}/\lambda_{inn,s}$. Substituting (A.1) and (A.2) gives (8). The partial elasticities in Appendix A.2 treat tariff-inclusive production-location costs as separate arguments; for a tariff change, wages and intermediate-input prices are held fixed.

A.2 Expenditure Share Elasticities

For $l, l' \neq n$, the conditional share ratio in (8) gives the within-technology-source trade elasticity:

$$\varepsilon_{n,s|i}^{ll'} \equiv \frac{\partial \ln(\lambda_{iln,s}/\lambda_{inn,s})}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = \begin{cases} -\dfrac{\theta_s}{1-\rho}, & l = l', \\ 0, & l \neq l', \end{cases} \tag{A.6}$$

This response is the same for every technology source i because the source-specific term $\tilde{\gamma}_{il,s}/\tilde{\gamma}_{in,s}$ in (8) does not change with the cost in l'.

The aggregate elasticity compares bilateral shares after summing over technology sources:

$$\lambda_{ln,s} = \frac{\sum_i D_{iln,s} D_{in,s}^{-\rho}}{D_{n,s}}, \qquad \frac{\lambda_{ln,s}}{\lambda_{nn,s}} = \frac{\sum_i D_{iln,s} D_{in,s}^{-\rho}}{\sum_i D_{inn,s} D_{in,s}^{-\rho}}. \tag{A.7}$$

Factoring the location costs out of (A.7) yields (11). To differentiate the ratio, (A.2) gives the following responses to the tariff-inclusive production cost $(1+t_{l'n,s})\tilde{\tau}_{l'n,s}$ in l':

$$\frac{\partial \ln D_{iln,s}}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = -\frac{\theta_s}{1-\rho}\mathbf{1}\{l=l'\}, \tag{A.8a}$$

$$\frac{\partial \ln D_{in,s}}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = -\frac{\theta_s}{1-\rho}\frac{D_{il'n,s}}{D_{in,s}}, \tag{A.8b}$$

$$\frac{\partial \ln D_{n,s}}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = -\theta_s \lambda_{l'n,s}. \tag{A.8c}$$

The response in (A.8c) follows from (A.8b), (7), and (10). Differentiating the numerator and denominator of (A.7) using (A.8a)–(A.8c) gives

$$\varepsilon^{ll'}_{n,s} \equiv \frac{\partial \ln(\lambda_{ln,s}/\lambda_{nn,s})}{\partial \ln[(1+t_{l'n,s})\tilde{\tau}_{l'n,s}]} = \varepsilon^{ll'}_{n,s|i} + \rho\frac{\theta_s}{1-\rho}\left[\sum_i \frac{\lambda_{iln,s}}{\lambda_{ln,s}}\frac{D_{il'n,s}}{D_{in,s}} - \sum_i \frac{\lambda_{inn,s}}{\lambda_{nn,s}}\frac{D_{il'n,s}}{D_{in,s}}\right]. \tag{A.9}$$

To express the source-composition response in (A.9) using expenditure shares, define the share of destination n's sectoral expenditure using technology from i:

$$\Lambda_{in,s} \equiv \sum_m \lambda_{imn,s}$$

Equation (7) then implies

$$\frac{D_{iln,s}}{D_{in,s}} = \frac{\lambda_{iln,s}}{\Lambda_{in,s}}, \qquad \lambda_{ln,s} = \sum_i \Lambda_{in,s}\frac{\lambda_{iln,s}}{\Lambda_{in,s}}, \qquad \Lambda_{in,s} = \frac{D^{1-\rho}_{in,s}}{D_{n,s}}. \tag{A.10}$$

Thus $\lambda_{ln,s}$ is the expenditure-weighted mean, across technology sources, of each source's fraction produced in l. Define the covariance of those production fractions for locations l and l' by

$$\mathrm{Cov}_{i\sim\Lambda_{\cdot n,s}}\left(\frac{\lambda_{iln,s}}{\Lambda_{in,s}}, \frac{\lambda_{il'n,s}}{\Lambda_{in,s}}\right) \equiv \sum_i \Lambda_{in,s}\left(\frac{\lambda_{iln,s}}{\Lambda_{in,s}} - \lambda_{ln,s}\right) \times \left(\frac{\lambda_{il'n,s}}{\Lambda_{in,s}} - \lambda_{l'n,s}\right). \tag{A.11}$$

Expanding (A.11) and using $\sum_i \Lambda_{in,s} = 1$ and (A.10) gives

$$\sum_i \frac{\lambda_{iln,s}}{\lambda_{ln,s}}\frac{D_{il'n,s}}{D_{in,s}} = \lambda_{l'n,s} + \frac{1}{\lambda_{ln,s}}\mathrm{Cov}_{i\sim\Lambda_{\cdot n,s}}\left(\frac{\lambda_{iln,s}}{\Lambda_{in,s}}, \frac{\lambda_{il'n,s}}{\Lambda_{in,s}}\right).$$

Substituting this identity into the bilateral-share derivative obtained from (A.7), and using the production-cost expression in (3), gives

$$\begin{aligned}\eta^{ll'}_{n,s} &\equiv \frac{\partial \ln \lambda_{ln,s}}{\partial \ln\left[(1+t_{l'n,s})w^{\alpha_{l',\ell s}}_{l'}\prod_k P^{\alpha_{l',ks}}_{l',k}\right]} \\ &= \frac{\theta_s}{1-\rho}\left[-\mathbf{1}\{l=l'\} + \lambda_{l'n,s} + \rho\frac{\mathrm{Cov}_{i\sim\Lambda_{\cdot n,s}}\left(\frac{\lambda_{iln,s}}{\Lambda_{in,s}}, \frac{\lambda_{il'n,s}}{\Lambda_{in,s}}\right)}{\lambda_{ln,s}}\right].\end{aligned} \tag{A.12}$$

A.3 Price Elasticity

Differentiating the sectoral price index in (4) with respect to $(1+t_{ln,s})u_{l,s}$ and using the share formula (7) gives the price response:

$$d\ln P_{n,s} = \sum_l \lambda_{ln,s}[d\ln(1+t_{ln,s}) + d\ln u_{l,s}].$$

From (3), $d\ln u_{l,s} = \alpha_{l,\ell s} d\ln w_l + \sum_k \alpha_{l,ks} d\ln P_{l,k}$. Substituting this expression shows both the initial tariff effect and its propagation through input costs:

$$d\ln P_{n,s} = \sum_l \lambda_{ln,s} d\ln(1+t_{ln,s}) + \sum_l \lambda_{ln,s}\alpha_{l,\ell s} d\ln w_l + \sum_{l,k} \underbrace{\lambda_{ln,s}\alpha_{l,ks}}_{\mathbf{O}_{(n,s),(l,k)}} d\ln P_{l,k}.$$

Because every sector uses labor, the row sum $\sum_{l,k}\mathbf{O}_{(n,s),(l,k)} = \sum_l \lambda_{ln,s}(1-\alpha_{l,\ell s})$ is strictly below one. Hence $\Psi = (I-\mathbf{O})^{-1} = I + \mathbf{O} + \mathbf{O}^2 + \cdots$. Holding wages fixed, a change in $t_{ld,k}$ creates a direct price change only at node (d,k), of size $\lambda_{ld,k} d\ln(1+t_{ld,k})$. Multiplying by Ψ yields the price elasticity in (36).

A.4 General Equilibrium Conditions

We collapse the general equilibrium conditions for quantitative implementation. Define

$$\begin{aligned}
\vartheta_{n,s} &\equiv \sum_l \frac{t_{ln,s}}{1+t_{ln,s}}\lambda_{ln,s}, & \zeta_n &\equiv 1 - \textstyle\sum_s \vartheta_{n,s}\mu_{n,s}, \\
\xi_{n,k} &\equiv \textstyle\sum_s \vartheta_{n,s}\alpha_{n,sk}, & \tilde{\alpha}_{ln,sk} &\equiv \dfrac{\lambda_{ln,s}}{1+t_{ln,s}}\alpha_{n,sk}, \\
\kappa_{ln,s} &\equiv \dfrac{\lambda_{ln,s}}{1+t_{ln,s}}\mu_{n,s}.
\end{aligned}$$

Using (13) and (15), final expenditure can be written as

$$X_n^f = \frac{w_n L_n + \sum_k \xi_{n,k} Y_{n,k}}{\zeta_n}.$$

Substituting this expression into goods-market clearing yields

$$Y_{l,s} = \sum_n \frac{\kappa_{ln,s}}{\zeta_n} w_n L_n + \sum_{n,k} \frac{\kappa_{ln,s}\xi_{n,k}}{\zeta_n} Y_{n,k} + \sum_{n,k} \tilde{\alpha}_{ln,sk} Y_{n,k}.$$

Substituting tariff revenue directly into sectoral expenditure gives

$$X_{n,s} = \mu_{n,s} w_n L_n + \mu_{n,s} \sum_k \vartheta_{n,k} X_{n,k} + \sum_k \alpha_{n,sk} Y_{n,k}.$$

A.5 Welfare and the Constrained Tariff Problem

Household preferences imply that welfare is real final consumption, $U_n = X_n^f / P_n$. Using the household budget constraint gives

$$U_n = \frac{w_n L_n + \mathrm{TR}_n}{P_n} = \frac{w_n L_n}{(1-\pi_n) P_n}, \qquad \pi_n \equiv \frac{\mathrm{TR}_n}{w_n L_n + \mathrm{TR}_n}. \tag{A.13}$$

Using (13) and (17), the tariff-revenue share can be expressed in equilibrium objects as

$$\pi_n = \frac{\sum_s \sum_l \dfrac{t_{ln,s}}{1+t_{ln,s}} \lambda_{ln,s} X_{n,s}}{\sum_s \alpha_{n,\ell s} Y_{n,s} + \sum_s \sum_l \dfrac{t_{ln,s}}{1+t_{ln,s}} \lambda_{ln,s} X_{n,s}}.$$

Let $\boldsymbol{t}_h \equiv \{t_{lh,s} : l \neq h,\ s = 1, \ldots, S\}$ be Home's vector of import tariffs, and let $\boldsymbol{t}_{-h} \equiv \{t_{ln,s} : n \neq h,\ l \neq n,\ s = 1, \ldots, S\}$ collect the tariffs set by all other destinations. Domestic tariffs satisfy $t_{nn,s} = 0$. The scalar $\bar{t} > 0$ is the upper bound on each admissible tariff. Home chooses $\boldsymbol{t}_h \in [0, \bar{t}]^{(N-1)S}$ while the other countries' tariff vector is fixed at $\bar{\boldsymbol{t}}_{-h}$. The full constrained optimization problem is

$$\max_{\boldsymbol{t}_h \in [0,\bar{t}]^{(N-1)S}} \ U_h = \frac{w_h L_h}{(1-\pi_h) P_h} \tag{A.14}$$

subject to $t = (\boldsymbol{t}_h, \bar{\boldsymbol{t}}_{-h})$ and the following constraints for $i, l, n = 1, \ldots, N$ and $s, k = 1, \ldots, S$:

$$c_{iln,s} = \tilde{\gamma}_{il,s} \tau_{ln,s} w_l^{\alpha_{l,\ell s}} \prod_k P_{l,k}^{\alpha_{l,ks}} (1 + t_{ln,s}), \tag{A.15}$$

$$\lambda_{iln,s} = \frac{c_{iln,s}^{-\frac{\theta_s}{1-\rho}} \left(\sum_m c_{imn,s}^{-\frac{\theta_s}{1-\rho}} \right)^{-\rho}}{\sum_j \left(\sum_m c_{jmn,s}^{-\frac{\theta_s}{1-\rho}} \right)^{1-\rho}}, \tag{A.16}$$

$$\lambda_{ln,s} = \sum_i \lambda_{iln,s}, \tag{A.17}$$

$$P_{n,s} = \tilde{\Gamma}_s \left\{ \sum_i \left[\sum_l c_{iln,s}^{-\frac{\theta_s}{1-\rho}} \right]^{1-\rho} \right\}^{-1/\theta_s}, \tag{A.18}$$

$$P_n = \prod_s (P_{n,s} / \mu_{n,s})^{\mu_{n,s}}, \tag{A.19}$$

$$Y_{l,s} = \sum_n \frac{\kappa_{ln,s}}{\zeta_n} w_n L_n + \sum_{n,k} \frac{\kappa_{ln,s} \xi_{n,k}}{\zeta_n} Y_{n,k} + \sum_{n,k} \tilde{\alpha}_{ln,sk} Y_{n,k}, \tag{A.20}$$

$$X_{n,s} = \mu_{n,s} w_n L_n + \mu_{n,s} \sum_k \vartheta_{n,k} X_{n,k} + \sum_k \alpha_{n,sk} Y_{n,k}, \tag{A.21}$$

$$w_l L_l = \sum_s \alpha_{l,\ell s} Y_{l,s}, \tag{A.22}$$

$$\sum_n w_n L_n = 1, \tag{A.23}$$

$$\pi_n = \frac{\sum_s \sum_l \frac{t_{ln,s}}{1+t_{ln,s}} \lambda_{ln,s} X_{n,s}}{\sum_s \alpha_{n,\ell s} Y_{n,s} + \sum_s \sum_l \frac{t_{ln,s}}{1+t_{ln,s}} \lambda_{ln,s} X_{n,s}}, \tag{A.24}$$

where

$$\begin{aligned}
\vartheta_{n,s} &\equiv \sum_l \frac{t_{ln,s}}{1+t_{ln,s}} \lambda_{ln,s}, & \zeta_n &\equiv 1 - \textstyle\sum_s \vartheta_{n,s} \mu_{n,s}, \\
\xi_{n,k} &\equiv \textstyle\sum_s \vartheta_{n,s} \alpha_{n,sk}, & \tilde{\alpha}_{ln,sk} &\equiv \frac{\lambda_{ln,s}}{1+t_{ln,s}} \alpha_{n,sk}, \\
\kappa_{ln,s} &\equiv \frac{\lambda_{ln,s}}{1+t_{ln,s}} \mu_{n,s}.
\end{aligned} \tag{A.25}$$

B Two-Country Optimal Tariffs

This appendix specializes the model from Section 1 to the two-country one-sector representation in Section 2 and proves all five propositions in Section 2.

B.1 The Two-Country, One-Sector Model

There are two countries h and f and one sector s. Set $\mu_{n,s} = 1$, $\alpha_{l,\ell s} = 1$, and all intermediate-input shares to zero. Labor supplies L_h and L_f are fixed, so $u_l = w_l$. Sector subscripts are suppressed, and $\theta > \sigma - 1$ ensures a finite CES price index. Domestic tariffs are zero, Foreign sets $t_{hf} = 0$, and Home chooses $t_{fh} \geq 0$.

For $i, l, n \in \{h, f\}$, the cost-adjusted technology terms are

$$D_{iln} = \left[\tilde{\gamma}_{il}\tau_{ln}(1 + t_{ln})w_l\right]^{-\frac{\theta}{1-\rho}}, \qquad D_{in} = \sum_{l\in\{h,f\}} D_{iln}, \qquad D_n = \sum_{i\in\{h,f\}} D_{in}^{1-\rho}. \tag{B.1}$$

Specializing (4) gives the price index

$$P_n = \tilde{\Gamma} D_n^{-1/\theta}. \tag{B.2}$$

With one sector, P_n is also the price index for final consumption. The expenditure shares are

$$\lambda_{iln} = \frac{D_{iln}D_{in}^{-\rho}}{D_n}, \qquad \lambda_{ln} = \sum_{i\in\{h,f\}} \lambda_{iln}. \tag{B.3}$$

Specializing tariff revenue in (13) gives $\mathrm{TR}_h = t_{fh}\lambda_{fh}X_h/(1 + t_{fh})$ and $\mathrm{TR}_f = 0$. Gross expenditure is final expenditure, so the household budgets are $X_n = w_nL_n + \mathrm{TR}_n$. Using $\lambda_{fh} = 1 - \lambda_{hh}$ in Home's budget yields

$$X_h = \frac{(1 + t_{fh})w_hL_h}{1 + t_{fh}\lambda_{hh}}, \qquad X_f = w_fL_f. \tag{B.4}$$

Labor is the only production input and profits are zero, so the value of goods produced in country l is w_lL_l. A destination spends $\lambda_{ln}X_n$ on goods produced in l, of which producers receive $\lambda_{ln}X_n/(1 + t_{ln})$ after the tariff. Goods and labor-market clearing therefore reduce to

$$w_hL_h = \lambda_{hh}X_h + \lambda_{hf}X_f, \qquad w_fL_f = \frac{\lambda_{fh}X_h}{1 + t_{fh}} + \lambda_{ff}X_f. \tag{B.5}$$

Substituting $X_f = w_fL_f$ into the Foreign condition and using $\lambda_{hf} = 1 - \lambda_{ff}$ equates Foreign export revenue to Foreign expenditure on Home production. The Home budget in (B.4) then gives trade balance at producer prices:

$$\lambda_{hf}w_fL_f = \frac{\lambda_{fh}X_h}{1 + t_{fh}} = \frac{\lambda_{fh}w_hL_h}{1 + t_{fh}\lambda_{hh}}. \tag{B.6}$$

The Foreign market-clearing condition in (B.5) also identifies the labor used to supply Home. Foreign uses $\lambda_{ff}X_f/w_f = \lambda_{ff}L_f$ units of labor to serve its domestic market. The labor remaining for goods sold in

Home is

$$L_f - \lambda_{ff} L_f = (1 - \lambda_{ff}) L_f = \lambda_{hf} L_f.$$

Trade balance (B.6) equates the wage bill for this production, $w_f \lambda_{hf} L_f$, to Home's expenditure on Foreign goods net of the tariff. Since L_f is fixed, the elasticity of Foreign labor used to supply Home with respect to w_f/w_h equals the elasticity of λ_{hf} with respect to w_f/w_h. This is η^X in (21).

Home's welfare is

$$U_h = \frac{w_h L_h}{P_h} \frac{1 + t_{fh}}{1 + t_{fh} \lambda_{hh}}. \tag{B.7}$$

Given a tariff, (B.1), (B.2), and (B.3) determine prices and shares from wages. Trade balance (B.6) determines the relative wage, while (18) determines the wage level. The budgets in (B.4) then determine gross expenditures. Together these equations form a reduced system for the competitive equilibrium.

B.2 Elasticities and Production-Location Choices

To measure how expenditure shares respond to a change in Foreign's production cost relative to Home's for destination n, hold Home's cost fixed. Because the shares are homogeneous in wages, varying w_f/w_h is equivalent to varying w_f with w_h fixed. Differentiating (B.1) with respect to the tariff-inclusive relative cost gives

$$\begin{aligned}
\frac{\partial \ln D_{iln}}{\partial \ln[(1 + t_{fn})(w_f/w_h)]} &= -\frac{\theta}{1 - \rho} \mathbf{1}\{l = f\}, \\
\frac{\partial \ln D_{in}}{\partial \ln[(1 + t_{fn})(w_f/w_h)]} &= -\frac{\theta}{1 - \rho} \frac{D_{ifn}}{D_{in}}, \\
\frac{\partial \ln D_{n}}{\partial \ln[(1 + t_{fn})(w_f/w_h)]} &= -\theta \lambda_{fn}.
\end{aligned} \tag{B.8}$$

The last equality follows from (B.3). Differentiating $\lambda_{hn} = \sum_i \lambda_{ihn}$ in (B.3) weights each source-specific response by $\lambda_{ihn}/\lambda_{hn}$ and gives

$$\frac{\partial \ln \lambda_{hn}}{\partial \ln[(1 + t_{fn})(w_f/w_h)]} = \theta \lambda_{fn} + \frac{\rho \theta}{1 - \rho} \sum_{i \in \{h,f\}} \frac{\lambda_{ihn}}{\lambda_{hn}} \frac{\lambda_{ifn}}{\lambda_{ihn} + \lambda_{ifn}}. \tag{B.9}$$

Set $n = h$ in (B.9). This yields η^M in (20). For Foreign's market, instead set $n = f$. Since $t_{hf} = t_{ff} = 0$, the relative cost changes only through w_f/w_h, so (B.9) yields η^X in (21). Using $\lambda_{iff}/(\lambda_{ihf} + \lambda_{iff}) = 1 - \lambda_{ihf}/(\lambda_{ihf} + \lambda_{iff})$ then gives (24).

The ratio $\lambda_{ihn}/(\lambda_{ihn} + \lambda_{ifn})$ is the fraction of country n's purchases made with technology from $i \in \{h, f\}$ that are produced in Home. The weight $\lambda_{ihn} + \lambda_{ifn}$ is the fraction of all purchases in n made with that technology. These weights sum to one, and their weighted mean Home-production fraction is

$$\sum_{i \in \{h,f\}} (\lambda_{ihn} + \lambda_{ifn}) \frac{\lambda_{ihn}}{\lambda_{ihn} + \lambda_{ifn}} = \lambda_{hn}.$$

The dispersion of the source-specific production fractions around the average across sources is thus

$$V_n = \sum_{i \in \{h,f\}} (\lambda_{ihn} + \lambda_{ifn}) \left(\frac{\lambda_{ihn}}{\lambda_{ihn} + \lambda_{ifn}} - \lambda_{hn} \right)^2. \tag{B.10}$$

Expanding the square gives

$$V_n = \sum_{i \in \{h,f\}} \frac{\lambda_{ihn}^2}{\lambda_{ihn} + \lambda_{ifn}} - \lambda_{hn}^2. \tag{B.11}$$

$$\begin{aligned} \lambda_{hn}\lambda_{fn} - V_n &= \sum_{i \in \{h,f\}} \left(\lambda_{ihn} - \frac{\lambda_{ihn}^2}{\lambda_{ihn} + \lambda_{ifn}} \right) \\ &= \sum_{i \in \{h,f\}} \frac{\lambda_{ihn}\lambda_{ifn}}{\lambda_{ihn} + \lambda_{ifn}} \geq 0, \\ 0 \leq V_n &\leq \lambda_{hn}\lambda_{fn}. \end{aligned} \tag{B.12}$$

The upper bound is approached when each technology source concentrates production in one location. Substituting (B.12) into (B.9) gives

$$\eta^M = \frac{\theta}{1-\rho} \left(\lambda_{fh} - \rho \frac{V_h}{\lambda_{hh}} \right), \qquad \eta^X = \frac{\theta}{1-\rho} \left(\lambda_{ff} - \rho \frac{V_f}{\lambda_{hf}} \right), \tag{B.13}$$

For either destination n, (B.3) implies $\lambda_{ihn}/\lambda_{ifn} = D_{ihn}/D_{ifn}$. Differentiating each technology source's Home-production fraction with respect to the tariff-inclusive relative cost therefore gives

$$\frac{\partial[\lambda_{ihn}/(\lambda_{ihn} + \lambda_{ifn})]}{\partial \ln[(1+t_{fn})(w_f/w_h)]} = \frac{\theta}{1-\rho} \frac{\lambda_{ihn}\lambda_{ifn}}{(\lambda_{ihn} + \lambda_{ifn})^2}. \tag{B.14}$$

The source-specific response in (B.14) is small when a technology source is concentrated in either location. Weighting these derivatives by each source's expenditure share, $\lambda_{ihn} + \lambda_{ifn}$, gives

$$\sum_{i \in \{h,f\}} (\lambda_{ihn} + \lambda_{ifn}) \frac{\partial[\lambda_{ihn}/(\lambda_{ihn} + \lambda_{ifn})]}{\partial \ln[(1+t_{fn})(w_f/w_h)]} = \frac{\theta}{1-\rho} (\lambda_{hn}\lambda_{fn} - V_n),$$

using (B.12). The second term in (B.9) is ρ/λ_{hn} times this weighted response. At given bilateral shares, a larger V_n means less adjustment in where each technology is used and therefore lowers both η^M and η^X in (B.13), for $n = h$ and $n = f$ respectively.

B.3 Welfare Derivative

Let $g = d\ln\bar{w}/d\ln(1+t_{fh})$ be the equilibrium response of Home's relative wage, as in Section 2.1. Since $\bar{w} = w_h/w_f$, the tariff-inclusive cost of Foreign production relative to Home production changes at rate $1-g$. To find the effect on Home's consumption cost in wage units, differentiate the price index in (B.2) and use the last derivative in (B.8) at $n = h$:

$$\frac{\partial \ln(P_h/w_h)}{\partial \ln[(1+t_{fh})(w_f/w_h)]} = -\frac{1}{\theta}\frac{\partial \ln D_h}{\partial \ln[(1+t_{fh})(w_f/w_h)]} = \lambda_{fh}.$$

Thus a one-percent increase in the tariff-inclusive relative cost of Foreign production raises P_h/w_h by λ_{fh} percent. The tariff changes that relative cost by $1-g$ percent. At $n = h$, (B.9) gives the response of λ_{hh}; $\lambda_{fh} = 1-\lambda_{hh}$ gives the response of Home's import share. At $n = f$, Home's tariff affects λ_{hf} only through the change $d\ln(w_f/w_h)/d\ln(1+t_{fh}) = -g$. The four total responses are

$$\frac{d\ln(P_h/w_h)}{d\ln(1+t_{fh})} = \lambda_{fh}(1-g), \tag{B.15a}$$

$$\frac{d\ln\lambda_{hh}}{d\ln(1+t_{fh})} = \eta^M(1-g), \tag{B.15b}$$

$$\frac{d\ln\lambda_{fh}}{d\ln(1+t_{fh})} = -\frac{\lambda_{hh}}{\lambda_{fh}}\eta^M(1-g), \tag{B.15c}$$

$$\frac{d\ln\lambda_{hf}}{d\ln(1+t_{fh})} = -\eta^X g. \tag{B.15d}$$

In Home's welfare expression (B.7), the tariff-revenue rebate enters through $1+t_{fh}\lambda_{hh}$. Both t_{fh} and λ_{hh} change with the tariff. Using $dt_{fh}/d\ln(1+t_{fh}) = 1+t_{fh}$ and (B.15b) gives

$$\frac{d\ln(1+t_{fh}\lambda_{hh})}{d\ln(1+t_{fh})} = \frac{\lambda_{hh}}{1+t_{fh}\lambda_{hh}}\left[1+t_{fh}+t_{fh}\eta^M(1-g)\right].$$

Log differentiation of (B.7), using (B.15a), therefore yields

$$\begin{aligned}\frac{d\ln U_h}{d\ln(1+t_{fh})} &= 1-\lambda_{fh}(1-g)-\frac{\lambda_{hh}[1+t_{fh}+t_{fh}\eta^M(1-g)]}{1+t_{fh}\lambda_{hh}}\\ &= \frac{\lambda_{fh}g - t_{fh}\lambda_{hh}(\lambda_{fh}+\eta^M)(1-g)}{1+t_{fh}\lambda_{hh}}.\end{aligned} \tag{B.16}$$

The numerator in the second line separates the two welfare effects in (22). The term $\lambda_{fh}g$ is the gain from Home's higher relative wage: Foreign goods become cheaper in units of Home income on the imports Home already buys. The term proportional to t_{fh} is the loss from the tariff wedge as Home's expenditure shifts away from Foreign production. The denominator $1+t_{fh}\lambda_{hh}$ comes from Home's household budget. This decomposition does not require a value for g.

To determine g, divide trade balance (B.6) by w_h:

$$\lambda_{hf}\frac{w_f}{w_h}L_f = \frac{\lambda_{fh}L_h}{1+t_{fh}\lambda_{hh}}.$$

Log differentiation shows each response before substitution:

$$\frac{d\ln\lambda_{hf}}{d\ln(1+t_{fh})}+\frac{d\ln(w_f/w_h)}{d\ln(1+t_{fh})}=\frac{d\ln\lambda_{fh}}{d\ln(1+t_{fh})}-\frac{d\ln(1+t_{fh}\lambda_{hh})}{d\ln(1+t_{fh})}.$$

The two terms on the left are $-\eta^X g$ by (B.15d) and $-g$ by the definition of g. The first term on the right is given by (B.15c); the second is the tariff-rebate derivative displayed above. Substitution and multiplication by -1 give

$$(1+\eta^X)g=\frac{\lambda_{hh}}{\lambda_{fh}}\eta^M(1-g)+\frac{\lambda_{hh}}{1+t_{fh}\lambda_{hh}}\left[1+t_{fh}+t_{fh}\eta^M(1-g)\right].$$

Rearranging gives

$$g=\frac{\lambda_{hh}(1+t_{fh})(\lambda_{fh}+\eta^M)}{\lambda_{hh}(1+t_{fh})(\lambda_{fh}+\eta^M)+\lambda_{fh}\left[\lambda_{fh}+(1+t_{fh}\lambda_{hh})\eta^X\right]}. \tag{B.17}$$

The shares and both elasticities are positive, so $0<g<1$: Home's relative wage rises, but by less than the tariff-inclusive import cost. Holding the tariff, bilateral shares, and the other elasticity fixed, g rises with η^M and falls with η^X. A larger shift in Home demand toward its own production raises its relative wage; a more elastic Foreign export supply requires a smaller wage change to restore trade balance.

B.4 Proof of Proposition 1

At an interior optimum, (B.16) gives the first-order condition

$$\lambda_{fh}g=t_{fh}\lambda_{hh}(\lambda_{fh}+\eta^M)(1-g). \tag{B.18}$$

Equation (B.17) implies

$$\frac{g}{1-g}=\frac{\lambda_{hh}(1+t_{fh})(\lambda_{fh}+\eta^M)}{\lambda_{fh}[\lambda_{fh}+(1+t_{fh}\lambda_{hh})\eta^X]}.$$

Substituting into (B.18) and cancelling positive factors gives

$$1+t_{fh}=t_{fh}[\lambda_{fh}+(1+t_{fh}\lambda_{hh})\eta^X],\qquad(1+t_{fh}\lambda_{hh})(1-t_{fh}\eta^X)=0.$$

Since (B.3) implies $\lambda_{hh}>0$ in equilibrium and $t_{fh}\geq 0$, the first factor is strictly positive. Hence $1-t_{fh}\eta^X=0$, or $t_{fh}=1/\eta^X$, as in (23). Setting $n=f$ in (B.9) gives the expression for η^X in (24).

Q.E.D.

B.5 Proof of Proposition 2

When multinational-production costs are prohibitive, technology from i can be used only in i, so only D_{iin} remains in each sum D_{in} in (B.1). Its contribution to D_n is

$$D_{iin}^{1-\rho}=[\tilde{\gamma}_{ii}\tau_{in}(1+t_{in})w_i]^{-\theta}=\left[\frac{\tau_{in}(1+t_{in})w_i}{T_{ii}}\right]^{-\theta}.$$

The power is θ: ρ drops out because there is no production-location choice within a technology source. The bilateral share follows from (B.3) with one surviving location for each source. In Foreign's market, where $t_{hf} = 0$, it is

$$\lambda_{hf}^{\text{nomp}} = \frac{(\tilde{\gamma}_{hh}\tau_{hf}w_h)^{-\theta}}{(\tilde{\gamma}_{hh}\tau_{hf}w_h)^{-\theta} + (\tilde{\gamma}_{ff}w_f)^{-\theta}}.$$

Differentiating this share with respect to the relative wage gives the no-multinational-production export-supply elasticity:

$$\eta^{X,\text{nomp}} \equiv \frac{\partial \ln \lambda_{hf}^{\text{nomp}}}{\partial \ln(w_f/w_h)} = \theta\lambda_{ff}^{\text{nomp}}. \tag{B.19}$$

At the matched bilateral shares, $\lambda_{ff}^{\text{nomp}} = \lambda_{ff}$.

Setting $n = f$ in (B.9) and subtracting (B.19) at the matched shares gives

$$\eta^X - \theta\lambda_{ff} = \frac{\rho\theta}{(1-\rho)\lambda_{hf}} \sum_{i\in\{h,f\}} \frac{\lambda_{ihf}\lambda_{iff}}{\lambda_{ihf} + \lambda_{iff}}.$$

With finite multinational-production costs and positive technology scales, $\lambda_{ihf} > 0$ and $\lambda_{iff} > 0$ for both $i = h$ and $i = f$. The right side is therefore positive when $\rho > 0$ and zero when $\rho = 0$, proving (25).

Q.E.D.

B.6 Proof of Proposition 3

In the trade-only model, Foreign's expenditure share on Home production is

$$\lambda_{hf}^{\text{trade}} = \frac{(\tau_{hf}^{\text{trade}}w_h/T_h^{\text{trade}})^{-\theta^{\text{trade}}}}{(\tau_{hf}^{\text{trade}}w_h/T_h^{\text{trade}})^{-\theta^{\text{trade}}} + (w_f/T_f^{\text{trade}})^{-\theta^{\text{trade}}}}. \tag{B.20}$$

Taking the log odds of (B.20) gives

$$\ln\frac{\lambda_{hf}^{\text{trade}}}{\lambda_{ff}^{\text{trade}}} = \theta^{\text{trade}} \ln\left(\frac{T_h^{\text{trade}}w_f}{\tau_{hf}^{\text{trade}}T_f^{\text{trade}}w_h}\right).$$

Differentiating the log odds with respect to the relative wage, and using $\lambda_{ff}^{\text{trade}} = 1 - \lambda_{hf}^{\text{trade}}$, yields its export-supply elasticity:

$$\eta^{X,\text{trade}} = \frac{\partial \ln \lambda_{hf}^{\text{trade}}}{\partial \ln(w_f/w_h)} = \theta^{\text{trade}}\lambda_{ff}^{\text{trade}}. \tag{B.21}$$

At the matched bilateral shares, $\lambda_{ff}^{\text{trade}} = \lambda_{ff}$. Under the parameter link $\theta^{\text{trade}} = \theta/(1-\rho)$ in (29), subtracting (B.21) from (28) gives

$$\eta^X - \eta^{X,\text{trade}} = \frac{\theta}{1-\rho}\left(\lambda_{ff} - \rho\frac{V_f}{\lambda_{hf}}\right) - \frac{\theta}{1-\rho}\lambda_{ff} = -\frac{\rho\theta}{1-\rho}\frac{V_f}{\lambda_{hf}}.$$

This proves (30). At fixed bilateral shares, the difference is more negative when production-location choices are more dispersed across technology sources, as measured by V_f. It is strictly negative when $\rho > 0$ and

$V_f > 0$.

Q.E.D.

B.7 Proof of Proposition 4

Now set $\theta^{\text{trade}} = \theta$ in (B.21). At the matched bilateral shares, the trade-only export-supply elasticity is $\eta^{X,\text{trade}} = \theta\lambda_{ff}$. Subtracting this from (28) yields

$$\eta^X - \eta^{X,\text{trade}} = \frac{\rho\theta}{1-\rho}\left(\lambda_{ff} - \frac{V_f}{\lambda_{hf}}\right) = \frac{\rho\theta}{(1-\rho)\lambda_{hf}} \sum_{i\in\{h,f\}} \frac{\lambda_{ihf}\lambda_{iff}}{\lambda_{ihf} + \lambda_{iff}},$$

where the second equality uses (B.12). Each term in the sum is positive with finite multinational-production costs and positive technology scales. Thus the difference is strictly positive when $\rho > 0$, proving (32).

Q.E.D.

B.8 Proof of Proposition 5

At common bilateral shares, (B.21) gives $\eta^{X,\text{trade}} = \theta^{\text{trade}}\lambda_{ff}$. Subtracting this from (28) and factoring out λ_{ff} gives

$$\eta^X - \eta^{X,\text{trade}} = \lambda_{ff}\left\{\frac{\theta}{1-\rho}\left[1 - \rho\frac{V_f}{\lambda_{hf}\lambda_{ff}}\right] - \theta^{\text{trade}}\right\}.$$

Since $\lambda_{ff} > 0$, the sign of the elasticity difference is the sign of the expression in braces, proving (33). Equation (B.12) implies $0 \le V_f/(\lambda_{hf}\lambda_{ff}) \le 1$. The threshold for θ^{trade} therefore equals $\theta/(1-\rho)$ when $V_f = 0$ and falls to θ at the upper bound for V_f. For intermediate dispersion, it lies between these two values.

Q.E.D.

C Additional Tables and Figures

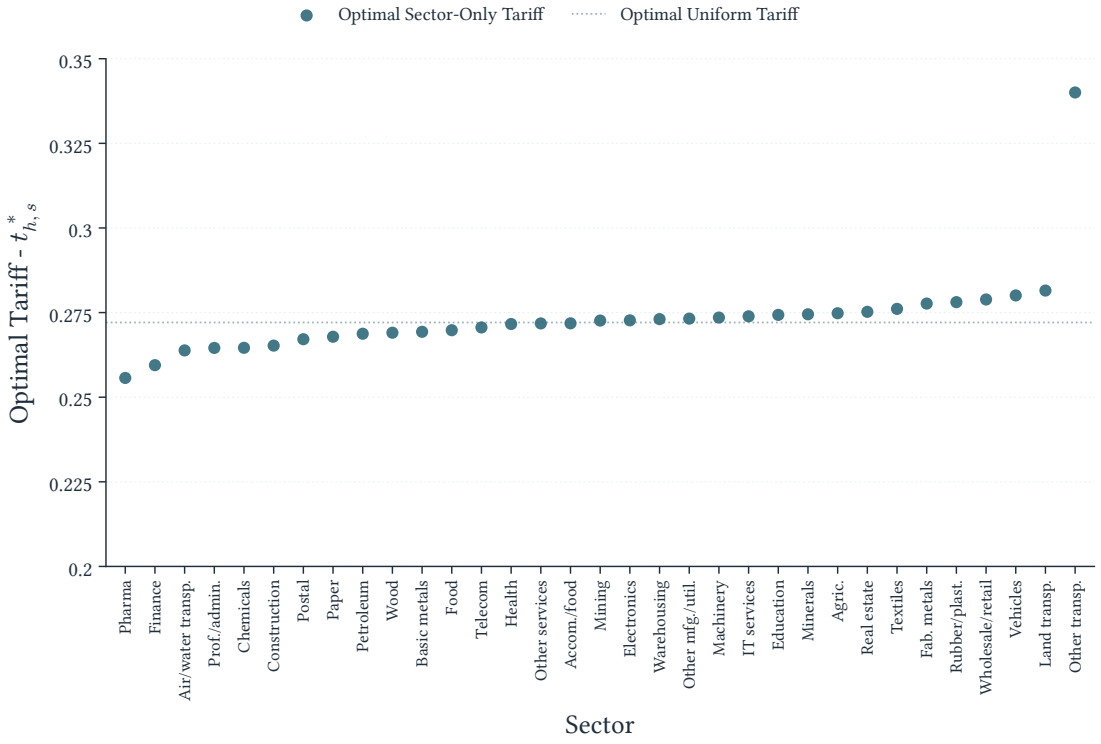


Figure C.1: Optimal U.S. Sector-Specific Tariffs in the MP Model

Notes: Each marker is an optimal sector-specific tariff, applied uniformly to all foreign origins within that sector. The dotted horizontal line marks the optimal uniform tariff. Foreign tariffs remain zero.

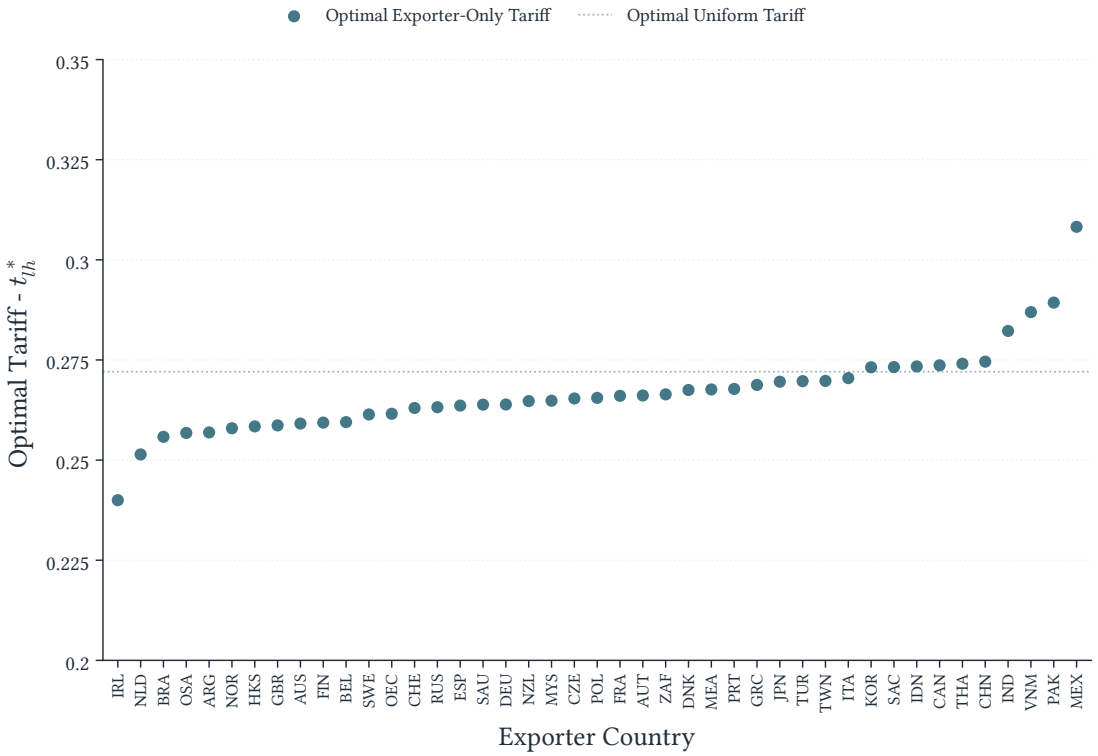


Figure C.2: Optimal U.S. Origin-Specific Tariffs in the MP Model

Notes: Each marker is an optimal origin-specific tariff, applied uniformly to all sectors from that origin. The dotted horizontal line marks the optimal uniform tariff. ROW is included in the tariff optimization but omitted from the figure. Foreign tariffs remain zero.

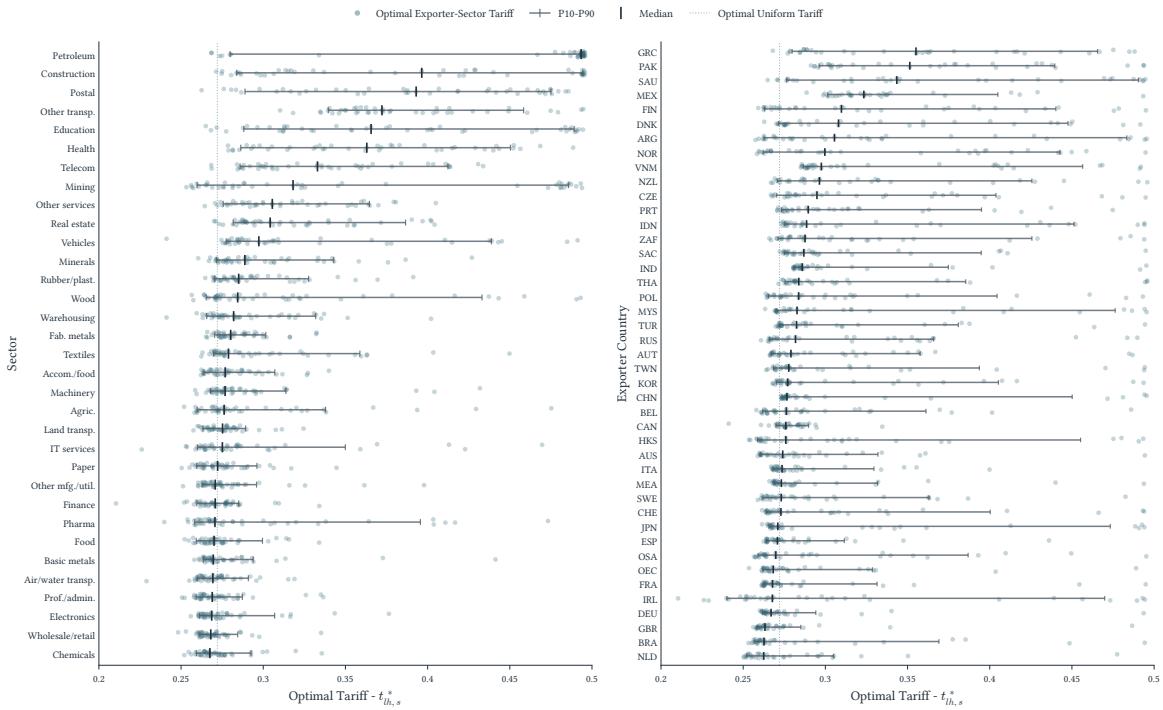


Figure C.3: Optimal U.S. Origin–Sector Tariffs in the MP Model

Notes: Dots are optimal origin–sector tariffs. The left panel groups tariffs by sector and the right panel by foreign origin. Horizontal segments span the 10th–90th percentiles, and vertical ticks mark medians. Dotted vertical lines mark the optimal uniform tariff. ROW is included in the tariff optimization but omitted from the figure.

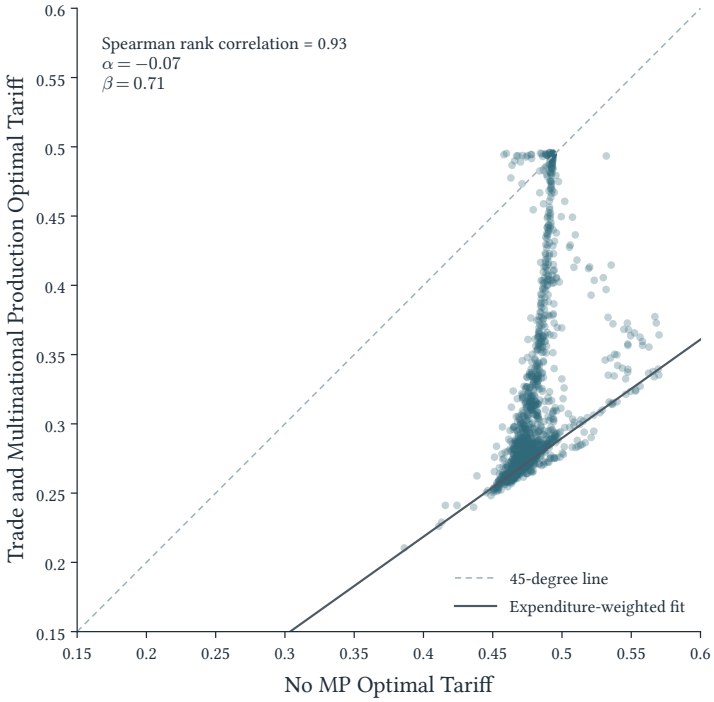


Figure C.4: Optimal U.S. country–sector tariffs with all sectors: comparing the MP and no-MP models

Notes: The comparison includes all 33 sectors, including mining and petroleum, and 43 foreign origins, covering 1,419 origin–sector pairs. ROW is included in the equilibrium and tariff optimization but omitted from the figure. The fitted line and Spearman rank correlation use baseline bilateral expenditure weights, $\lambda_{lh,s} X_{h,s}$. The corresponding main-text comparison excludes mining and petroleum. In the figure, α is the intercept and β is the slope of the fitted line.

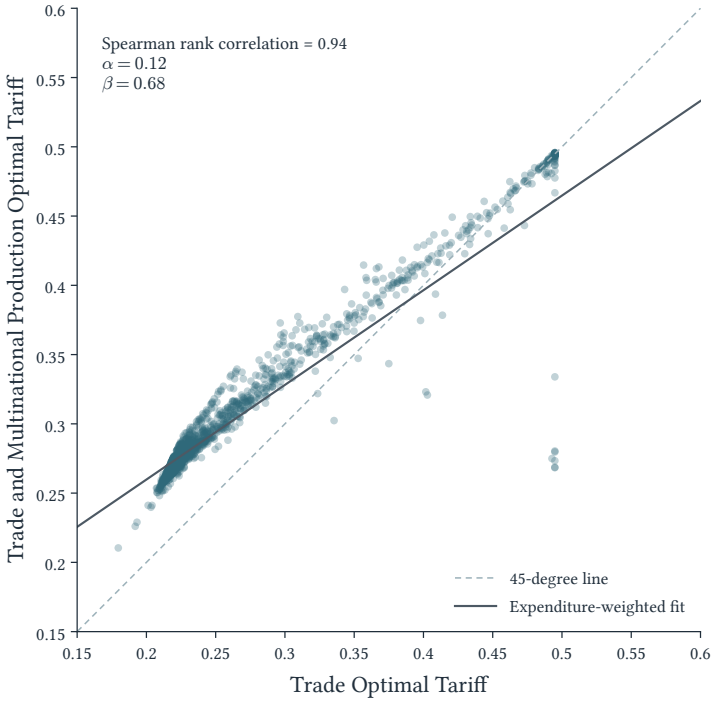


Figure C.5: Optimal U.S. country–sector tariffs with all sectors: comparing the MP and trade models

Notes: The comparison includes all 33 sectors, including mining and petroleum, and 43 foreign origins, covering 1,419 origin–sector pairs. ROW is included in the equilibrium and tariff optimization but omitted from the figure. The fitted line and Spearman rank correlation use baseline bilateral expenditure weights, $\lambda_{lh,s} X_{h,s}$. The corresponding main-text comparison excludes mining and petroleum. In the figure, α is the intercept and β is the slope of the fitted line.

Table C.1: Largest Tariff Deviations Across Starts in the MP Model

Exporter–Sector		Run 1	Run 2	Run 3	Run 4	Run 5
CAN —	Petroleum	47.41×10^{-2}	48.26×10^{-2}	48.84×10^{-2}	27.49×10^{-2}	48.05×10^{-2}
BEL —	Petroleum	26.98×10^{-2}	26.80×10^{-2}	43.58×10^{-2}	26.86×10^{-2}	26.86×10^{-2}
FRA —	Petroleum	39.31×10^{-2}	40.02×10^{-2}	40.04×10^{-2}	27.97×10^{-2}	28.40×10^{-2}
PRT —	Petroleum	36.65×10^{-2}	37.22×10^{-2}	36.90×10^{-2}	27.34×10^{-2}	27.21×10^{-2}
ITA —	Petroleum	40.86×10^{-2}	41.55×10^{-2}	41.62×10^{-2}	33.39×10^{-2}	32.04×10^{-2}

Note: Results use 2020 baseline data for USA. Rows show the 5 exporter-sectors with the largest maximum absolute tariff deviation across all pairwise comparisons, in descending order.

Table C.2: Largest Tariff Deviations Across Starts in the No-MP Model

Exporter–Sector		Run 1	Run 2	Run 3	Run 4	Run 5
VNM —	Other transp.	52.73×10^{-2}	53.59×10^{-2}	53.55×10^{-2}	53.65×10^{-2}	53.55×10^{-2}
SAC —	Other transp.	52.21×10^{-2}	53.04×10^{-2}	52.99×10^{-2}	53.06×10^{-2}	53.00×10^{-2}
IDN —	Other transp.	50.32×10^{-2}	51.13×10^{-2}	50.96×10^{-2}	51.09×10^{-2}	51.04×10^{-2}
DNK —	Other transp.	50.13×10^{-2}	50.93×10^{-2}	50.75×10^{-2}	50.89×10^{-2}	50.84×10^{-2}
MYS —	Other transp.	51.30×10^{-2}	52.10×10^{-2}	52.00×10^{-2}	52.10×10^{-2}	52.01×10^{-2}

Note: Results use 2020 baseline data for USA. Rows show the 5 exporter-sectors with the largest maximum absolute tariff deviation across all pairwise comparisons, in descending order.

Table C.3: Largest Tariff Deviations Across Starts in the Trade-Only Model

Exporter–Sector		Run 1	Run 2	Run 3	Run 4	Run 5
ESP —	Mining	32.84×10^{-2}	24.90×10^{-2}	32.92×10^{-2}	35.50×10^{-2}	25.42×10^{-2}
SWE —	Mining	26.67×10^{-2}	24.96×10^{-2}	26.98×10^{-2}	33.76×10^{-2}	24.82×10^{-2}
TUR —	Mining	33.84×10^{-2}	25.89×10^{-2}	33.92×10^{-2}	33.99×10^{-2}	25.79×10^{-2}
JPN —	Mining	36.39×10^{-2}	29.17×10^{-2}	36.44×10^{-2}	35.02×10^{-2}	30.73×10^{-2}
ITA —	Mining	38.68×10^{-2}	33.27×10^{-2}	38.72×10^{-2}	39.64×10^{-2}	32.99×10^{-2}

Note: Results use 2020 baseline data for USA. Rows show the 5 exporter-sectors with the largest maximum absolute tariff deviation across all pairwise comparisons, in descending order.

D Data

This appendix documents the cleaning of the data and the construction of the baseline used in Section 4. I first reconcile the production and use accounts and construct labor cost shares, intermediate-input shares, final-demand shares, and bilateral expenditures. I then combine ownership-production output with bilateral sales to infer trilateral flows under the model's share restrictions. Finally, I construct a balanced baseline and describe the sectoral elasticity inputs. The baseline uses 2020 observations from the 2024 AAMNE release, aggregated to the 45 countries and 33 sectors in Tables D.2 and D.1.

The superscript raw denotes a reported observation. Output, expenditure, and share objects marked data denote the final model-consistent baseline. Unmarked flows refer to the cleaned accounts before the balancing step; they are inputs to the calibration and need not yet satisfy all equilibrium conditions. A tilde denotes a provisional value within a cleaning step. Throughout, i indexes the country that owns the technology, l the production country, and n the destination. In an intermediate flow, the first sector index denotes the input sector and the second the using sector, consistently with $\alpha_{n,sk}$ in Section 1.

D.1 Production, Consumption, and Bilateral Trade Flows

The 2024 AAMNE release is harmonized with the OECD inter-country input–output tables at basic prices (Cadestin et al. 2018). Transaction values exclude taxes less subsidies on products, including import tariffs. Trade and transport margins are recorded in the corresponding service sectors. The ownership-output tables are reconciled with these production accounts, so their totals can be combined with bilateral sales. I interpret the data as a zero-tariff baseline.

Let $\mathrm{IO}^{\mathrm{raw}}_{ln,sk}$ denote sales of sector-s goods produced in l for intermediate use by sector k in n. Let $X^{f,\mathrm{raw}}_{ln,s}$ denote final sales and $\mathrm{INV}^{\mathrm{raw}}_{ln,s}$ inventory changes. Under zero tariffs, producer-value sales and tariff-inclusive expenditure coincide. Reported gross output, gross value added, and the valuation adjustment are denoted by $\mathrm{GO}^{\mathrm{raw}}_{n,k}$, $\mathrm{GVA}^{\mathrm{raw}}_{n,k}$, and $\mathrm{TAXSUB}^{\mathrm{raw}}_{n,k}$. The production and use accounts satisfy

$$\mathrm{GO}^{\mathrm{raw}}_{n,k} = \mathrm{GVA}^{\mathrm{raw}}_{n,k} + \mathrm{TAXSUB}^{\mathrm{raw}}_{n,k} + \sum_{l,s} \mathrm{IO}^{\mathrm{raw}}_{ln,sk},$$

$$\mathrm{GO}^{\mathrm{raw}}_{l,s} = \sum_{n} \left(\sum_{k} \mathrm{IO}^{\mathrm{raw}}_{ln,sk} + X^{f,\mathrm{raw}}_{ln,s} + \mathrm{INV}^{\mathrm{raw}}_{ln,s} \right).$$

During the cleaning steps, gross output is held at its reported value, $Y_{n,s} \equiv \mathrm{GO}^{\mathrm{raw}}_{n,s}$.

Value added and labor cost shares. The valuation adjustment reconciles the cost and use valuations in the AAMNE accounts. It is not an observed bilateral tariff and is therefore included in value added:

$$\mathrm{VA}^{\mathrm{raw}}_{n,s} \equiv \mathrm{GVA}^{\mathrm{raw}}_{n,s} + \mathrm{TAXSUB}^{\mathrm{raw}}_{n,s} = Y_{n,s} - \sum_{l,r} \mathrm{IO}^{\mathrm{raw}}_{ln,rs}.$$

Because labor is the model's only factor of production, the value-added share is mapped to the labor cost share. Some country–sector observations have nonpositive value added even after aggregation. Rather than aggregate further and lose sectoral information, I replace these shares using valid observations for

the same country–sector pair in other years.

For year t, define $\alpha_{n,\ell s}^{\mathrm{raw},t} \equiv \mathrm{VA}_{n,s}^{\mathrm{raw},t}/Y_{n,s}^{t}$. Positive observations are retained. For an isolated nonpositive observation bracketed by valid adjacent years, set

$$\alpha_{n,\ell s}^{t} = \frac{1}{2}\left(\alpha_{n,\ell s}^{\mathrm{raw},t-1} + \alpha_{n,\ell s}^{\mathrm{raw},t+1}\right).$$

If only one adjacent year is valid, use its share. For a consecutive spell of at most five nonpositive observations, let t_0 and t_1 be the nearest valid years before and after the spell and interpolate:

$$\alpha_{n,\ell s}^{t} = \alpha_{n,\ell s}^{\mathrm{raw},t_0} + \frac{t-t_0}{t_1-t_0}\left(\alpha_{n,\ell s}^{\mathrm{raw},t_1} - \alpha_{n,\ell s}^{\mathrm{raw},t_0}\right), \qquad t_0 < t < t_1.$$

The year superscript is suppressed below, where all objects refer to 2020.

Intermediate inputs and the cost identity. After correcting the labor share, set $\mathrm{VA}_{n,k} = \alpha_{n,\ell k} Y_{n,k}$. Rescale intermediate purchases proportionately so that their total equals $(1-\alpha_{n,\ell k})Y_{n,k}$:

$$\nu_{n,k}^{\mathrm{IO}} \equiv \frac{(1-\alpha_{n,\ell k})Y_{n,k}}{\sum_{l,s}\mathrm{IO}_{ln,sk}^{\mathrm{raw}}}, \qquad \mathrm{IO}_{ln,sk} \equiv \nu_{n,k}^{\mathrm{IO}}\mathrm{IO}_{ln,sk}^{\mathrm{raw}}.$$

This preserves the composition of input purchases across origins and sectors. The intermediate-input shares are then

$$\alpha_{n,sk} \equiv \frac{\sum_l \mathrm{IO}_{ln,sk}}{Y_{n,k}}, \qquad \alpha_{n,\ell k} + \sum_s \alpha_{n,sk} = 1.$$

Final sales and bilateral expenditure. The model has no inventory accumulation. After adjusting intermediate purchases, I allocate the residual output available for final use across destinations in proportion to reported final sales:

$$\nu_{l,s}^{f} \equiv \frac{Y_{l,s} - \sum_{n,k}\mathrm{IO}_{ln,sk}}{\sum_n X_{ln,s}^{f,\mathrm{raw}}}, \qquad \widetilde{X}_{ln,s}^{f} \equiv \nu_{l,s}^{f} X_{ln,s}^{f,\mathrm{raw}}.$$

The numerator is positive at the chosen aggregation. This step absorbs inventory changes and the use-side discrepancy induced by the cost adjustment into final sales. Let $\widetilde{B}_{ln,s} \equiv \sum_k \mathrm{IO}_{ln,sk} + \widetilde{X}_{ln,s}^{f}$ denote provisional bilateral sales at producer value. Some of these flows are zero, whereas finite costs and positive technology scales imply positive bilateral shares in the model.

For origin-sector (l,s), let $\mathcal{N}_{l,s}^{f} \equiv \sum_n \mathbf{1}\{\widetilde{B}_{ln,s} = 0\}$. If $\mathcal{N}_{l,s}^{f} > 0$, choose a small $\epsilon_{l,s}^{f} > 0$ satisfying $\epsilon_{l,s}^{f}\mathcal{N}_{l,s}^{f} < \sum_n \widetilde{X}_{ln,s}^{f}$ and set

$$X_{ln,s}^{f} \equiv \begin{cases} \epsilon_{l,s}^{f}, & \widetilde{B}_{ln,s} = 0, \\ \widetilde{X}_{ln,s}^{f}\dfrac{\sum_m \widetilde{X}_{lm,s}^{f} - \epsilon_{l,s}^{f}\mathcal{N}_{l,s}^{f}}{\sum_m \widetilde{X}_{lm,s}^{f}}, & \widetilde{B}_{ln,s} > 0. \end{cases}$$

If there are no zero bilateral flows, set $X^f_{ln,s} = \widetilde{X}^f_{ln,s}$. The adjustment preserves total final sales from each origin-sector and hence the use-side identity $Y_{l,s} = \sum_n (\sum_k \mathrm{IO}_{ln,sk} + X^f_{ln,s})$.

Bilateral expenditure and destination–sector totals are

$$X_{ln,s} \equiv (1 + t_{ln,s}) \left(\sum_k \mathrm{IO}_{ln,sk} + X^f_{ln,s} \right), \qquad X_{n,s} \equiv \sum_l X_{ln,s}, \qquad \lambda_{ln,s} \equiv \frac{X_{ln,s}}{X_{n,s}}.$$

The baseline has $t_{ln,s} = 0$, so producer-value sales and tariff-inclusive expenditure coincide. Final-demand shares are consequently

$$\mu_{n,s} \equiv \frac{\sum_l X^f_{ln,s}}{\sum_{l,r} X^f_{ln,r}}, \qquad \sum_s \mu_{n,s} = 1.$$

D.2 Multinational Production and Trilateral Trade Flows

Ownership-production output. Let $Y^{\mathrm{MP,raw}}_{il,s}$ denote output produced in country l and sector s using technology owned in i. The ownership data satisfy $\sum_i Y^{\mathrm{MP,raw}}_{il,s} = Y_{l,s}$, but some source–location flows are zero. To obtain positive ownership margins without changing aggregate output, define $\mathcal{N}_{l,s} \equiv \sum_i \mathbf{1}\{Y^{\mathrm{MP,raw}}_{il,s} = 0\}$. For $\mathcal{N}_{l,s} > 0$, choose a small $\epsilon_{l,s} > 0$ satisfying $\epsilon_{l,s}\mathcal{N}_{l,s} < Y_{l,s}$ and set

$$Y^{\mathrm{MP}}_{il,s} \equiv \begin{cases} \epsilon_{l,s}, & Y^{\mathrm{MP,raw}}_{il,s} = 0, \\ Y^{\mathrm{MP,raw}}_{il,s} \dfrac{Y_{l,s} - \epsilon_{l,s}\mathcal{N}_{l,s}}{Y_{l,s}}, & Y^{\mathrm{MP,raw}}_{il,s} > 0. \end{cases}$$

Otherwise, retain the reported values. This preserves $\sum_i Y^{\mathrm{MP}}_{il,s} = Y_{l,s}$ while making every ownership-production margin positive.

Recovering trilateral expenditure. Ownership-production output is valued before tariffs, whereas bilateral expenditure includes tariffs. The trilateral shares must therefore satisfy both margin restrictions:

$$\begin{aligned} Y^{\mathrm{MP}}_{il,s} &= \sum_n \frac{\lambda_{iln,s} X_{n,s}}{1 + t_{ln,s}}, \\ X_{ln,s} &= \sum_i \lambda_{iln,s} X_{n,s}. \end{aligned} \tag{D.1}$$

These restrictions alone do not identify the destination of each source–location's output. I also impose the separable cost structure in (A.1) and the sourcing formula (7). For fixed ρ, I jointly adjust the positive source–location cost components $\tilde{\gamma}_{il,s} u_{l,s}$ and location–destination trade costs $\tau_{ln,s}$ until the shares reproduce both margins in (D.1). The resulting flows are uniquely determined by the positive margins and ρ, even though the cost components need not be uniquely determined.

To establish this distinction, consider the zero-tariff baseline and fix sector s. The share formula in (7)

implies

$$(1-\rho)\ln\lambda_{iln,s}+\rho\ln\left(\sum_m \lambda_{imn,s}\right)=-\theta_s\ln(\tilde{\gamma}_{il,s}u_{l,s})-\theta_s\ln\tau_{ln,s}-\ln D_{n,s},$$

where $D_{n,s}$ is defined in (A.2). Thus, the left side separates into a source–location component and a location–destination component. This is also the first-order condition, with constants absorbed into the multipliers on the two margins, for

$$\min_{\{X_{iln,s}\geq 0\}}(1-\rho)\sum_{i,l,n}X_{iln,s}\ln\left(\frac{X_{iln,s}}{X_{n,s}}\right)+\rho\sum_{i,n}\left(\sum_l X_{iln,s}\right)\ln\left(\frac{\sum_l X_{iln,s}}{X_{n,s}}\right),$$

subject to $\sum_n X_{iln,s}=Y^{\mathrm{MP}}_{il,s}$ and $\sum_i X_{iln,s}=X_{ln,s}$, with $X_{n,s}$ fixed. Positive, consistent margins admit a strictly positive feasible allocation, for example $X_{iln,s}=Y^{\mathrm{MP}}_{il,s}X_{ln,s}/Y_{l,s}$. For $0\leq\rho<1$, the first term in the objective is strictly convex in the trilateral flows and the second is convex. The objective therefore has a unique minimizer; its marginal derivative at a zero flow ensures that the minimizer is interior. Its first-order conditions are precisely the separability restriction implied by the model. Hence any positive model allocation matching both margins must coincide with this unique solution.

The recovered expenditure flows are $X_{iln,s}=\lambda_{iln,s}X_{n,s}$. They match the cleaned margins and obey the model's share restrictions. Their trilateral decomposition depends on ρ and is model-imputed; it is not an additional set of independently observed trade flows.

D.3 Constructing the Balanced Baseline

The cleaning and reconstruction steps preserve the production and use identities, but the resulting accounts can contain country-level trade imbalances. I remove these imbalances before using the accounts as the baseline for tariff counterfactuals, following Costinot and Rodríguez-Clare (2014). I first express all reference value flows in units such that world labor income is one, $\sum_{n,s}\alpha_{n,\ell s}Y_{n,s}=1$. The cleaned shares, output, and labor incomes then provide the reference values for the price and sourcing equations in exact changes. I solve for the equilibrium with no deficit transfers, keeping tariffs at $\boldsymbol{t}'=\boldsymbol{t}=\mathbf{0}$ and holding $\alpha_{n,\ell s}$, $\alpha_{n,sk}$, and $\mu_{n,s}$ fixed. The level conditions are stated in Appendix A.5, and their exact-change representation is given in Appendix E.1.

Relative wages and sectoral prices adjust to clear markets. These changes alter sourcing shares, gross output, and expenditure even though tariffs do not change. Denoting this equilibrium by primes, the final baseline objects are

$$Y^{\mathrm{data}}_{n,s}\equiv Y'_{n,s},\qquad X^{\mathrm{data}}_{n,s}\equiv X'_{n,s},\qquad \lambda^{\mathrm{data}}_{iln,s}\equiv\lambda'_{iln,s}.$$

Their bilateral aggregates and expenditure flows are

$$\lambda^{\mathrm{data}}_{ln,s}\equiv\sum_i\lambda^{\mathrm{data}}_{iln,s},\quad X^{\mathrm{data}}_{ln,s}\equiv\lambda^{\mathrm{data}}_{ln,s}X^{\mathrm{data}}_{n,s},\quad X^{\mathrm{data}}_{iln,s}\equiv\lambda^{\mathrm{data}}_{iln,s}X^{\mathrm{data}}_{n,s}.$$

These objects satisfy the full equilibrium conditions, including balanced trade and the normalization of world labor income. The superscript data thus identifies the adjusted baseline reproduced by the calibration, rather than the original raw observations. The trade-only model in Section 4.4 is calibrated to the same final bilateral shares and production and expenditure accounts.

D.4 Sectoral Elasticity Inputs

Table D.1 maps the AAMNE sectors into the 33 model sectors and reports the empirical elasticity inputs $\varepsilon_s^{\text{data}}$ used in (42). The column labeled CP15 gives the corresponding sector codes in Caliendo and Parro (2015). When more than one of their estimates maps to a model sector, I take the unweighted average. Sectors without a corresponding goods-sector estimate are assigned a value of five; these entries are calibration assumptions rather than separate empirical estimates. Table D.2 reports the country aggregation, including the composite regions and rest of the world.

The table reports empirical elasticity inputs rather than the endogenous aggregate elasticities of the full model. Their mapping into the parameters of the two models is described in Sections 4.2 and 4.4.

Table D.1: Sectoral aggregations

Sector	Description	AAMNE	CP15	$\frac{\theta_s}{1-\rho}$
1	Agriculture, forestry and fishing	A01T03	1	8.110
2	Mining and extraction of energy producing products	B05T09	2	15.720
3	Food products, beverages and tobacco	C10T12	3	2.550
4	Textiles, wearing apparel, leather and related products	C13T15	4	5.560
5	Wood and products of wood and cork	C16	5	10.830
6	Paper products and printing; publishing, audiovisual, and broadcasting activities	C17T18, J58T60	6	9.070
7	Coke and refined petroleum products	C19	7	51.080
8	Chemicals and chemical products;	C20	8	4.750
9	Pharmaceuticals, medicinal chemical and botanical products	C21	8	4.750
10	Rubber and plastic products	C22	9	1.660
11	Other non-metallic mineral products	C23	10	2.760
12	Basic metals	C24	11	7.990
13	Fabricated metal products	C25	12	4.300
14	Computer, electronic and optical products; electrical equipment	C26, C27	14, 15, 16, 17	9.860
15	Machinery and equipment, n.e.c.	C28	13, 18	1.265
16	Motor vehicles, trailers and semi-trailers	C29	18	1.010
17	Other transport equipment	C30	19	0.370
18	Other manufacturing; repair and installation of machinery and equipment; utilities	C31T33, D35_E36T39	20, 21	5.000
19	Construction	F41T43	22	5.000
20	Wholesale and retail trade; repair of motor vehicles	G45T47	23	5.000
21	Land transport and transport via pipelines	H49	25	5.000
22	Water and air transport	H50, H51	26, 27	5.000
23	Warehousing and support activities for transportation	H52	28	5.000
24	Postal and courier activities	H53	29	5.000
25	Accommodation and food services	I55T56	24	5.000
26	Telecommunications	J61	29	5.000
27	IT and other information services	J62T63	33	5.000
28	Financial and insurance activities	K64T66	30	5.000
29	Real estate activities	L68	31	5.000

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Table D.1: Sectoral aggregations (continued)

Sector	Description	AAMNE	CP15	$\frac{\theta_s}{1-\rho}$
30	Professional, scientific and technical activities; administrative and support services; public admin. and defence	M69T75, N77T82, O84	32, 34, 35, 36	5.000
31	Education	P85	37	5.000
32	Human health and social work	Q86T88	38	5.000
33	Arts, entertainment and recreation; other service activities; private households with employed persons	R90T93, S94T96, T97T98	39, 40	5.000

Table D.2: Country aggregations

Code	Name	AAMNE Countries
ARG	Argentina	ARG
AUS	Australia	AUS
AUT	Austria	AUT
BEL	Belgium	BEL
BRA	Brazil	BRA
CAN	Canada	CAN
CHE	Switzerland	CHE
CHN	China (People's Republic of)	CHN
CZE	Czechia	CZE
DEU	Germany	DEU
DNK	Denmark	DNK
ESP	Spain	ESP
FIN	Finland	FIN
FRA	France	FRA
GBR	United Kingdom	GBR
GRC	Greece	GRC
IDN	Indonesia	IDN
IND	India	IND
IRL	Ireland	IRL
ITA	Italy	ITA
JPN	Japan	JPN
KOR	Korea	KOR
MEX	Mexico	MEX
MYS	Malaysia	MYS
NLD	Netherlands	NLD
NOR	Norway	NOR
NZL	New Zealand	NZL
PAK	Pakistan	PAK
POL	Poland	POL
PRT	Portugal	PRT
RUS	Russian Federation	RUS
SAU	Saudi Arabia	SAU
SWE	Sweden	SWE
THA	Thailand	THA
TUR	Türkiye	TUR

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Table D.2: Country aggregations (continued)

Code	Name	AAMNE Countries
TWN	Chinese Taipei	TWN
USA	United States	USA
VNM	Viet Nam	VNM
ZAF	South Africa	ZAF
HKS	Hong Kong and Singapore	HKG, SGP
OSA	Other South America	CHL, COL, PER
OEC	Other European Countries	BGR, BLR, CYP, EST, HRV, HUN, ISL, LTU, LUX, LVA, MLT, ROU, SVK, SVN, UKR
MEA	Middle East and Africa	CIV, CMR, EGY, ISR, JOR, MAR, NGA, SEN, TUN
SAC	Small Asian Countries	BRN, KHM, LAO, MMR, PHL
ROW	Rest of the World	BGD, CRI, KAZ, ROW

E Solving for Optimal Tariffs

This appendix develops the equilibrium and derivative results used to solve Home's quantitative optimal-tariff problem. It proceeds in four steps. First, the counterfactual equilibrium is written in exact changes relative to a fixed baseline, an exact-hat labor-market residual is defined, and equilibrium wage changes are characterized as a normalized fixed point. Second, the appendix shows that maximizing Home welfare U_h in exact changes yields the same tariff choice as maximizing its level. Third, it differentiates the price, trade-share, gross-output, gross-expenditure, and tariff-revenue blocks with respect to wages and tariffs. Finally, it differentiates equilibrium wages and combines the block derivatives with an adjoint method to compute the full welfare gradient without constructing the policy Jacobian of every intermediate equilibrium object. Appendices F–H collect the supporting lemmas, exact-hat algebra, and detailed derivative calculations.

E.1 Solving for the equilibrium

Assumption 1 collects the regularity conditions used to characterize and compute the equilibrium. Part (i) restricts policy to admissible nonnegative tariffs and sets domestic tariffs to zero. Parts (ii)–(iii) guarantee that the sectoral CES price moments exist and rule out zero or infinite productivity primitives. Part (iv) rules out zero or infinite trade and multinational-production costs and imposes the triangle inequality. Part (v) ensures that each sector enters final demand with strictly positive weight. Part (vi) imposes nonnegative input shares and requires labor to be used in every sector. Part (vii) is an interiority condition. Strict positivity of $Y_{n,s}$, $X_{n,s}$, and $\lambda_{iln,s}$ ensures that the exact-hat representation and the derivatives used below are well defined. Since bilateral trade shares aggregate trilateral shares according to $\lambda_{ln,s} = \sum_i \lambda_{iln,s}$, strict positivity of $\lambda_{iln,s}$ implies $\lambda_{ln,s} > 0$ for all (l, n, s). Thus every country sources a strictly positive share of each sector from every origin. This rules out disconnected trade blocks and ensures that shocks to demand or trade costs can propagate through the global production network.

Assumption 1 (Model parameters and equilibrium objects) *For all country and sector indices:*

(i) Fix $\bar{t} \in (0, 1)$ and define the admissible tariff set

$$\mathbf{T} \equiv \left\{ \boldsymbol{t} \in \mathbb{R}^{N \times N \times S} : t_{ln,s} \in [0, \bar{t}] \text{ for } l \neq n, \quad t_{nn,s} = 0 \right\}.$$

(ii) Sectoral CES elasticities satisfy $\sigma_s \in (1, \infty)$.

(iii) Fréchet parameters satisfy $\rho \in [0, 1)$, $\theta_s > \sigma_s - 1$, and $T_{il,s} \in (0, \infty)$.

(iv) Trade and multinational-production costs are finite and satisfy $\tau_{ln,s} \geq 1$, $\gamma_{il,s} \geq 1$, $\tau_{nn,s} = 1$, and $\gamma_{ii,s} = 1$, with triangle inequalities $\tau_{ln,s} \leq \tau_{lm,s}\tau_{mn,s}$ and $\gamma_{il,s} \leq \gamma_{im,s}\gamma_{ml,s}$.

(v) Final expenditure shares satisfy $\mu_{n,s} \in (0, 1]$ and

$$\sum_s \mu_{n,s} = 1.$$

(vi) Production cost shares satisfy $\alpha_{l,\ell s} \in [\bar{\alpha}, 1]$ where $\bar{\alpha} > 0$, $\alpha_{l,ks} \in [0, 1 - \bar{\alpha}]$, and

$$\alpha_{l,\ell s} + \sum_k \alpha_{l,ks} = 1.$$

(vii) Equilibrium quantities and expenditure shares are interior $Y_{n,s} > 0$, $X_{n,s} > 0$, and $\lambda_{iln,s} \in (0, 1]$.

Because domestic tariffs satisfy $t_{nn,s} = 0$, they are fixed rather than policy variables. The following convention is therefore adopted throughout the appendix.

Definition 1 (Continuously differentiable in tariffs) *A variable is said to be* continuously differentiable in tariffs *if it has continuous partial derivatives with respect to the non-domestic tariffs $t_{ln,s}$, $l \neq n$, while domestic tariffs are held fixed at zero. At the bounds 0 and $\bar{t}$, derivatives are understood as the corresponding one-sided limits. To keep the notation readable, the full tariff vector $\boldsymbol{t}$ continues to be used rather than introducing a separate vector containing only its non-domestic components. Jacobians stored with the full tariff vector have zero columns for the fixed domestic tariffs.*

The model is represented using exact-hat algebra to minimize the number of parameters that need to be calibrated. Let $z(\boldsymbol{t})$ denote a *baseline* equilibrium under tariffs $\boldsymbol{t}$, and let $z(\boldsymbol{t}')$ denote a *counterfactual* equilibrium under tariffs $\boldsymbol{t}'$. For each strictly positive endogenous object x, write x for its baseline value, x' for its counterfactual value, and define $\hat{x} \equiv x'/x$. Because the maintained parameter restrictions ensure that the baseline denominators are positive, this division is well-defined and yields a closed system in the hat variables that depends on $(\boldsymbol{t}', \boldsymbol{t})$, parameters $\{\theta_s\}_{s=1}^{S}$ and ρ, labor shares $\alpha_{l,\ell s}$, intermediate-input shares $\alpha_{l,ks}$, final-consumption shares $\mu_{n,s}$, and baseline equilibrium values $\lambda_{iln,s}$, $Y_{n,s}$, and $X_{n,s}$. Importantly, the baseline values are themselves equilibrium values and therefore satisfy all level equilibrium conditions, including labor-market clearing and the chosen normalization

$$1 = \sum_{n,s} \alpha_{n,\ell s} Y_{n,s}. \tag{E.1}$$

Equations (E.2)–(E.8), together with the accounting definitions in (E.9), form the exact-hat system for the counterfactual equilibrium, with the baseline equilibrium held fixed. The cost $c_{iln,s}$ is the normalized cost in (A.15). Its exact change is independent of technology source i, so $\hat{c}_{iln,s} = \hat{c}_{ln,s}$, as derived in Appendix G.

$$\hat{c}_{ln,s} = \frac{1 + t'_{ln,s}}{1 + t_{ln,s}} \left(\hat{w}_l^{\alpha_{l,\ell s}} \prod_k \hat{P}_{l,k}^{\alpha_{l,ks}} \right), \tag{E.2}$$

$$\hat{\lambda}_{iln,s} = \hat{c}_{ln,s}^{-\frac{\theta_s}{1-\rho}} \frac{\left(\sum_m \frac{\lambda_{imn,s}}{\Lambda_{in,s}} \hat{c}_{mn,s}^{-\frac{\theta_s}{1-\rho}} \right)^{-\rho}}{\sum_j \Lambda_{jn,s} \left(\sum_m \frac{\lambda_{jmn,s}}{\Lambda_{jn,s}} \hat{c}_{mn,s}^{-\frac{\theta_s}{1-\rho}} \right)^{1-\rho}}, \tag{E.3}$$

$$\hat{\lambda}_{ln,s} = \sum_i \frac{\lambda_{iln,s}}{\lambda_{ln,s}} \hat{\lambda}_{iln,s}, \tag{E.4}$$

$$\hat{P}_{n,s} = \left(\sum_i \Lambda_{in,s} \left(\sum_l \frac{\lambda_{iln,s}}{\Lambda_{in,s}} \hat{c}_{ln,s}^{-\frac{\theta_s}{1-\rho}} \right)^{1-\rho} \right)^{-\frac{1}{\theta_s}}, \tag{E.5}$$

$$Y'_{l,s} = \sum_{n,k} \frac{\kappa'_{ln,s}}{\zeta'_n} \alpha_{n,\ell k} Y'_{n,k} + \sum_{n,k} \frac{\kappa'_{ln,s} \xi'_{n,k}}{\zeta'_n} Y'_{n,k} + \sum_{n,k} \tilde{\alpha}'_{ln,sk} Y'_{n,k}, \tag{E.6}$$

$$\hat{w}_n = \frac{\sum_s \alpha_{n,\ell s} Y'_{n,s}}{\sum_k \alpha_{n,\ell k} Y_{n,k}}, \tag{E.7}$$

$$1 = \sum_{n,s} \alpha_{n,\ell s} Y_{n,s} \hat{w}_n. \tag{E.8}$$

The counterfactual accounting objects are

$$\begin{aligned} \vartheta'_{n,s} &\equiv \sum_l \frac{t'_{ln,s}}{1+t'_{ln,s}} \lambda'_{ln,s}, & \zeta'_n &\equiv 1 - \sum_s \vartheta'_{n,s} \mu_{n,s}, \\ \xi'_{n,k} &\equiv \sum_s \vartheta'_{n,s} \alpha_{n,sk}, & \tilde{\alpha}'_{ln,sk} &\equiv \frac{\lambda'_{ln,s}}{1+t'_{ln,s}} \alpha_{n,sk}, \\ \kappa'_{ln,s} &\equiv \frac{\lambda'_{ln,s}}{1+t'_{ln,s}} \mu_{n,s}. \end{aligned} \tag{E.9}$$

Given the equilibrium variables in changes, the aggregate price indices, gross expenditure, and welfare are recovered using

$$\hat{P}_n = \prod_s (\hat{P}_{n,s})^{\mu_{n,s}}, \tag{E.10}$$

$$X'_{n,s} = \sum_k \left(\mu_{n,s}\alpha_{n,\ell k} + \alpha_{n,sk}\right) Y'_{n,k} + \mu_{n,s} \sum_k \vartheta'_{n,k} X'_{n,k}, \tag{E.11}$$

$$\hat{U}_n = \frac{\hat{w}_n}{\hat{P}_n} \times \frac{1-\pi_n}{1-\pi'_n}, \tag{E.12}$$

where the counterfactual tariff-revenue share is

$$\pi'_n = \frac{\sum_s \vartheta'_{n,s} X'_{n,s}}{\sum_s \alpha_{n,\ell s} Y'_{n,s} + \sum_s \vartheta'_{n,s} X'_{n,s}}. \tag{E.13}$$

Appendix G provides additional details on deriving (E.3)–(E.13).

Following Alvarez and Lucas (2007), the counterfactual equilibrium is characterized by its country-level labor-market imbalances. Fix a baseline tariff vector $\boldsymbol{t} \in \mathbf{T}$ and a strictly positive baseline equilibrium $z(\boldsymbol{t})$. For a counterfactual tariff vector $\boldsymbol{t}' \in \mathbf{T}$ and a candidate wage-change vector $\hat{\boldsymbol{w}}$, the exact-hat equations (E.3) – (E.6), (E.9), and (E.11) determine $\hat{\boldsymbol{P}}$, $\hat{\boldsymbol{\lambda}}$, $\boldsymbol{Y}'$, and $\boldsymbol{X}'$ as functions of $(\hat{\boldsymbol{w}}, \boldsymbol{t}')$.

Define country n's baseline labor income by

$$b_n \equiv w_n L_n = \sum_s \alpha_{n,\ell s} Y_{n,s} > 0, \qquad \sum_n b_n = 1. \tag{E.14}$$

For a candidate normalized wage-change vector, the *exact-hat labor-market residual* is

$$\hat{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv \sum_s \alpha_{n,\ell s} Y'_{n,s}(\hat{\boldsymbol{w}}, \boldsymbol{t}') - b_n \hat{w}_n. \tag{E.15}$$

Here $\hat{Z}_n$ denotes the labor-income residual in levels constructed from the exact-hat blocks. The admissible

normalized wage changes are the relative interior of the weighted simplex

$$\Delta_b \equiv \left\{ \hat{\boldsymbol{w}} \in \mathbb{R}^N_{++} : \boldsymbol{b}^\top \hat{\boldsymbol{w}} = 1 \right\}, \tag{E.16}$$

where $\boldsymbol{b} \equiv (b_1, \ldots, b_N)^\top$.

For the numerical wage update, let $\hat{\boldsymbol{Z}} \equiv (\hat{Z}_1, \ldots, \hat{Z}_N)^\top$ and define the scaling matrix and the projection onto the tangent space of the normalization by

$$\mathbf{D}_Y \equiv \operatorname{diag}\left(\frac{1}{\sum_s Y_{1,s}}, \ldots, \frac{1}{\sum_s Y_{N,s}} \right), \qquad \Pi_b \equiv \mathbf{I}_N - \frac{\boldsymbol{b}\boldsymbol{b}^\top}{\boldsymbol{b}^\top \boldsymbol{b}}. \tag{E.17}$$

The scaled and projected residual is

$$\tilde{\boldsymbol{Z}}(\hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv \Pi_b \mathbf{D}_Y \hat{\boldsymbol{Z}}(\hat{\boldsymbol{w}}, \boldsymbol{t}'). \tag{E.18}$$

Fix $\beta > 0$. For each $\boldsymbol{t}' \in \mathbf{T}$, define the wage-update map $\mathcal{W}(\cdot, \boldsymbol{t}') : \Delta_b \to \Delta_b$ componentwise by

$$\mathcal{W}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv \frac{\hat{w}_n \exp\left[\beta \tilde{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}')\right]}{\sum_m b_m \hat{w}_m \exp\left[\beta \tilde{Z}_m(\hat{\boldsymbol{w}}, \boldsymbol{t}')\right]}. \tag{E.19}$$

Scaling mitigates the effect of differences in country size on the numerical update, and projection places the scaled residual in the tangent space of the normalization. The denominator in (E.19) ensures that updated wages remain in Δ_b.

Proposition E.1 establishes that an exact-hat equilibrium exists for every admissible counterfactual tariff vector and that solving the labor-market residual system together with the wage normalization is equivalent to finding a fixed point of the normalized wage-update map. I do not establish uniqueness of the equilibrium, which is beyond the scope of the paper.

Proposition E.1 (Existence of an exact-hat equilibrium) *Suppose Assumption 1 holds. Fix a baseline tariff vector $\boldsymbol{t} \in \mathbf{T}$ and a strictly positive baseline equilibrium $z(\boldsymbol{t})$ satisfying (E.1). For every counterfactual tariff vector $\boldsymbol{t}' \in \mathbf{T}$, there exists at least one normalized wage-change vector $\hat{\boldsymbol{w}}^* \in \Delta_b$ and associated strictly positive exact changes satisfying (E.2)–(E.11) such that*

$$\hat{Z}_n(\hat{\boldsymbol{w}}^*, \boldsymbol{t}') = 0 \qquad \textit{for every } n.$$

Equivalently, $\hat{\boldsymbol{w}}^*$ is an interior fixed point of $\mathcal{W}(\cdot, \boldsymbol{t}')$. Recovering counterfactual levels by $x' = \hat{x}\, x$ yields a competitive equilibrium under $\boldsymbol{t}'$. The residual mappings $\hat{\boldsymbol{Z}}$ and $\tilde{\boldsymbol{Z}}$, and the wage-update map $\mathcal{W}$, are continuously differentiable in the counterfactual tariffs in the sense of Definition 1.

Proof. For every $\hat{\boldsymbol{w}} \in \mathbb{R}^N_{++}$, define the wage-scaled labor excess-demand function

$$\mathcal{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv \frac{\boldsymbol{b}^\top \hat{\boldsymbol{w}}}{\hat{w}_n} \sum_s \alpha_{n,\ell s} Y'_{n,s}(\hat{\boldsymbol{w}}, \boldsymbol{t}') - b_n.$$

To solve for a unique gross-output vector for any candidate normalized wage vector, the normalization in

(E.27) is imposed. The factor $\boldsymbol{b}^\top \hat{\boldsymbol{w}}$ restores the nominal scale removed by this normalization and thereby defines $\mathcal{Z}_n$ for every strictly positive wage vector $\hat{\boldsymbol{w}} \in \mathbb{R}^N_{++}$, rather than only for normalized wages in Δ_b. In particular, if $\hat{\boldsymbol{w}} \in \Delta_b$, then $\boldsymbol{b}^\top \hat{\boldsymbol{w}} = 1$ and

$$\mathcal{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}') = \frac{\hat{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}')}{\hat{w}_n}.$$

Thus $\mathcal{Z}_n$ and $\hat{Z}_n$ have the same zeros on the normalized wage simplex.

Fix $\boldsymbol{t}' \in \mathbf{T}$. The proof follows Alvarez and Lucas (2007, Theorem 2), who invoke the general excess-demand existence theorem in Mas-Colell, Whinston, and Green (1995, Proposition 17.C.1). To apply that theorem, it is sufficient to show that $\boldsymbol{\mathcal{Z}}$ is continuous, homogeneous of degree zero, satisfies Walras' law, is uniformly bounded below, and diverges in at least one component when wages approach a nonzero boundary point of $\mathbb{R}^N_+$.

Continuity. Propositions E.3, E.4, E.5, and E.6 establish that the conditional price, trade-share, normalized gross-output, and gross-expenditure mappings are continuous and strictly positive. Lemma 4 ensures that the accounting denominators used by these mappings are bounded away from zero. Because every $\hat{w}_n$ is positive, the displayed definition of $\mathcal{Z}_n$ is therefore continuous on $\mathbb{R}^N_{++}$.

Homogeneity of degree zero. Let $a > 0$. Constant returns to scale and uniqueness of the price solution imply $\hat{\boldsymbol{P}}(a\hat{\boldsymbol{w}}, \boldsymbol{t}') = a\hat{\boldsymbol{P}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$. Delivered costs therefore all scale by a, leaving the trade shares unchanged. Since the normalized output mapping depends on wages through these trade shares,

$$\boldsymbol{Y}'(a\hat{\boldsymbol{w}}, \boldsymbol{t}') = \boldsymbol{Y}'(\hat{\boldsymbol{w}}, \boldsymbol{t}').$$

It follows directly that

$$\begin{aligned}\mathcal{Z}_n(a\hat{\boldsymbol{w}}, \boldsymbol{t}') &= \frac{\boldsymbol{b}^\top(a\hat{\boldsymbol{w}})}{a\hat{w}_n} \sum_s \alpha_{n,\ell s} Y'_{n,s}(a\hat{\boldsymbol{w}}, \boldsymbol{t}') - b_n \\ &= \frac{a\boldsymbol{b}^\top \hat{\boldsymbol{w}}}{a\hat{w}_n} \sum_s \alpha_{n,\ell s} Y'_{n,s}(\hat{\boldsymbol{w}}, \boldsymbol{t}') - b_n = \mathcal{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}').\end{aligned}$$

Thus a common rescaling of all wages does not change labor excess demand.

Walras' law. Let

$$\mathcal{L}_n(\hat{\boldsymbol{w}}) \equiv \sum_s \alpha_{n,\ell s} Y'_{n,s}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$$

denote country n's labor income in the normalized output vector. Equation (E.27) gives $\sum_n \mathcal{L}_n(\hat{\boldsymbol{w}}) = 1$. Multiplying each country's excess demand by its wage and summing gives

$$\begin{aligned}\sum_n \hat{w}_n \mathcal{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}') &= (\boldsymbol{b}^\top \hat{\boldsymbol{w}}) \sum_n \mathcal{L}_n(\hat{\boldsymbol{w}}) - \sum_n b_n \hat{w}_n \\ &= \boldsymbol{b}^\top \hat{\boldsymbol{w}} - \boldsymbol{b}^\top \hat{\boldsymbol{w}} = 0.\end{aligned}$$

This is Walras' law: the value of world labor excess demand is zero at every positive wage vector.

Uniform lower bound. Assumption 1 and Proposition E.5 imply $Y'_{n,s} \geq 0$, while $\alpha_{n,\ell s} \geq 0$, $\boldsymbol{b}^\top \hat{\boldsymbol{w}} > 0$, and $\hat{w}_n > 0$. The first term in the definition of $\mathcal{Z}_n$ is therefore nonnegative. Consequently,

$$\mathcal{Z}_n(\hat{\boldsymbol{w}}, \boldsymbol{t}') \geq -b_n \geq -\max_m b_m \qquad \text{for every } n \text{ and } \hat{\boldsymbol{w}} \in \mathbb{R}^N_{++}.$$

Boundary behavior and existence. Lemma 5 establishes the required boundary behavior on Δ_b. For any positive wage sequence approaching a nonzero boundary point, dividing every vector by $\boldsymbol{b}^\top \hat{\boldsymbol{w}}$ places the sequence in Δ_b without changing $\boldsymbol{\mathcal{Z}}$, by degree-zero homogeneity. The rescaled sequence converges to a nonzero boundary point because $\boldsymbol{b} \gg \mathbf{0}$. The lemma therefore implies that at least one component of labor excess demand diverges to $+\infty$.

All five hypotheses of Mas-Colell, Whinston, and Green (1995, Proposition 17.C.1) now hold, so there exists $\hat{\boldsymbol{w}}^\dagger \in \mathbb{R}^N_{++}$ such that $\mathcal{Z}_n(\hat{\boldsymbol{w}}^\dagger, \boldsymbol{t}') = 0$ for every n. Define $\hat{\boldsymbol{w}}^* \equiv \hat{\boldsymbol{w}}^\dagger / (\boldsymbol{b}^\top \hat{\boldsymbol{w}}^\dagger)$. Then $\hat{\boldsymbol{w}}^* \in \Delta_b$, and degree-zero homogeneity implies $\boldsymbol{\mathcal{Z}}(\hat{\boldsymbol{w}}^*, \boldsymbol{t}') = \mathbf{0}$. On Δ_b, $\mathcal{Z}_n = \hat{Z}_n / \hat{w}_n$; hence $\hat{Z}_n(\hat{\boldsymbol{w}}^*, \boldsymbol{t}') = 0$ for every n. At this wage vector, Propositions E.3, E.4, E.5, and E.6 define the associated equilibrium prices, trade shares, gross outputs, and gross expenditures conditional on wages. Together with $\hat{\boldsymbol{Z}}(\hat{\boldsymbol{w}}^*, \boldsymbol{t}') = \mathbf{0}$, which supplies the remaining labor-market equations, these objects satisfy the full exact-hat equilibrium system.

Fixed-point equivalence. If $\hat{\boldsymbol{Z}} = \mathbf{0}$, then $\tilde{\boldsymbol{Z}} = \mathbf{0}$ and the denominator in (E.19) is $\sum_n b_n \hat{w}_n = 1$, so $\mathcal{W}(\hat{\boldsymbol{w}}, \boldsymbol{t}') = \hat{\boldsymbol{w}}$. Conversely, suppose $\mathcal{W}(\hat{\boldsymbol{w}}, \boldsymbol{t}') = \hat{\boldsymbol{w}}$ for some $\hat{\boldsymbol{w}} \in \Delta_b$. Since every $\hat{w}_n > 0$, (E.19) implies that $\exp(\beta \tilde{Z}_n)$ is the same for every n. Thus $\tilde{\boldsymbol{Z}} = c\mathbf{1}$ for some scalar c. But $\boldsymbol{b}^\top \tilde{\boldsymbol{Z}} = 0$ and $\sum_n b_n = 1$, so $c = 0$. The identity $\Pi_b \mathbf{D}_Y \hat{\boldsymbol{Z}} = \mathbf{0}$ then implies $\mathbf{D}_Y \hat{\boldsymbol{Z}} = d\boldsymbol{b}$ for some scalar d. Finally,

$$\sum_n \hat{Z}_n = \sum_n \mathcal{L}_n(\hat{\boldsymbol{w}}) - \sum_n b_n \hat{w}_n = 0.$$

Since $\mathbf{D}_Y^{-1} \boldsymbol{b} \gg \mathbf{0}$, this equality forces $d = 0$, and therefore $\hat{\boldsymbol{Z}} = \mathbf{0}$.

Regularity. Propositions E.3, E.4, E.5, and E.6 establish continuous differentiability of all conditional equilibrium blocks. The residual mappings and $\mathcal{W}$ are compositions of these mappings with sums, products, exponentials, and strictly positive denominators. They are therefore continuously differentiable in the non-domestic tariffs, with the one-sided interpretation at tariff bounds in Definition 1.

Q.E.D.

E.2 Exact-Hat Optimal Tariff Problem

Since the main object of this paper is to choose tariffs that maximize welfare, it is necessary to ensure that optimizing in the exact-hat representation delivers the same optimal tariff schedule as optimizing in levels. Proposition E.2 establishes this result using the same convention as the equilibrium system: $\boldsymbol{t}$ denotes the fixed reference tariff vector, $\boldsymbol{t}'$ denotes a candidate counterfactual tariff vector, and hats denote the ratio of the counterfactual equilibrium to the reference equilibrium.

Proposition E.2 *Suppose Assumption 1 holds. Fix an exogenous tariff schedule $\bar{\boldsymbol{t}}_{-h}$ and a reference Home tariff vector $\bar{\boldsymbol{t}}_h$ such that $\boldsymbol{t} = (\bar{\boldsymbol{t}}_h, \bar{\boldsymbol{t}}_{-h}) \in \mathbf{T}$. Let $z(\boldsymbol{t})$ denote the equilibrium under $\boldsymbol{t}$. For each candidate $\boldsymbol{t}_h \in [0, \bar{t}]^{(N-1)S}$, let $\boldsymbol{t}' = (\boldsymbol{t}_h, \bar{\boldsymbol{t}}_{-h})$ and let $z(\boldsymbol{t}')$ denote a selected counterfactual equilibrium, using the same selection in the level and exact-hat formulations. Define*

$$\hat{x}(\boldsymbol{t}', \boldsymbol{t}) \equiv \frac{x(\boldsymbol{t}')}{x(\boldsymbol{t})} \quad \text{for each strictly positive endogenous object } x, \qquad \hat{U}_h(\boldsymbol{t}', \boldsymbol{t}) \equiv \frac{U_h(z(\boldsymbol{t}'), \boldsymbol{t}')}{U_h(z(\boldsymbol{t}), \boldsymbol{t})}.$$

Then the level and exact-hat formulations have the same set of optimal unilateral tariffs:

$$\operatorname*{arg\,max}_{\boldsymbol{t}_h \in [0,\bar{t}]^{(N-1)S}} \log U_h(z(\boldsymbol{t}'), \boldsymbol{t}') = \operatorname*{arg\,max}_{\boldsymbol{t}_h \in [0,\bar{t}]^{(N-1)S}} \log \hat{U}_h(\boldsymbol{t}', \boldsymbol{t}).$$

Proof. Fix the reference equilibrium $z(\boldsymbol{t})$. For any candidate $\boldsymbol{t}_h \in [0, \bar{t}]^{(N-1)S}$, equation (E.12) gives

$$\log \hat{U}_h(\boldsymbol{t}', \boldsymbol{t}) = \log U_h(z(\boldsymbol{t}'), \boldsymbol{t}') - \log U_h(z(\boldsymbol{t}), \boldsymbol{t}).$$

The second term is finite and constant in the candidate $\boldsymbol{t}_h$. Hence the two objectives differ by an additive constant on the same feasible set $[0, \bar{t}]^{(N-1)S}$ and therefore have the same $\arg\max$ set.

Q.E.D.

E.3 Derivatives of Equilibrium Objects

Fix the baseline tariff vector $\boldsymbol{t} = (\bar{\boldsymbol{t}}_h, \bar{\boldsymbol{t}}_{-h}) \in \mathbf{T}$ and denote the tariff vector under the counterfactual $\boldsymbol{t}_h$ as $\boldsymbol{t}' = (\boldsymbol{t}_h, \bar{\boldsymbol{t}}_{-h}) \in \mathbf{T}$. Throughout this section, all mappings are written as functions of $(\hat{\boldsymbol{w}}, \boldsymbol{t}')$, with the understanding that parameters and baseline objects (including $\boldsymbol{t}$) enter as constants.

Vectors are stacked across countries and sectors. For any variable x, boldface denotes its stacked vector, a prime denotes a counterfactual level, and a hat denotes an Exact-Hat change. Objects indexed by (n, s) are stacked sector-fastest in $\mathbb{R}^{NS}$, so element $e = (n-1)S + s$ corresponds to (n, s). Objects indexed by (l, n, s) are stacked in $\mathbb{R}^{NNS}$, so element $e = (l-1)NS + (n-1)S + s$ corresponds to (l, n, s). Let e_j denote the relevant standard basis vector. For a tuple index, write $e_{(n,s)}$ or $e_{(l,n,s)}$ for the basis vector associated with that stacked index.

For any vector-valued map $\boldsymbol{x}(\boldsymbol{y})$, write $\boldsymbol{x}_{\boldsymbol{y}} \equiv \partial \boldsymbol{x} / \partial \boldsymbol{y}$, so that $\boldsymbol{x}_{\boldsymbol{y}}$ has rows indexed by components of $\boldsymbol{x}$ and columns indexed by components of $\boldsymbol{y}$. Let $(\boldsymbol{x}_{\boldsymbol{y}})_{(a;b)}$ denote $\partial x_a / \partial y_b$, where $(a; \cdot)$ and $(\cdot; b)$ collect row a and column b of $\boldsymbol{x}_{\boldsymbol{y}}$, respectively.

E.3.1 Price Indices

Given $(\hat{\boldsymbol{w}}, \boldsymbol{t}')$, the sectoral price changes in (E.2) and (E.5) are determined independently of the remaining equilibrium objects. Define $\hat{p}_{n,s} \equiv \ln \hat{P}_{n,s}$ and stack these log price changes in $\hat{\boldsymbol{p}} \in \mathbb{R}^{NS}$. The price system is the fixed point $\hat{\boldsymbol{p}} = \mathcal{P}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{w}}, \boldsymbol{t}')$, where

$$\mathcal{P}_{n,s}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv -\frac{1}{\theta_s} \ln \left[\sum_i \Lambda_{in,s} \left(\sum_l \frac{\lambda_{iln,s}}{\Lambda_{in,s}} \hat{c}_{ln,s}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{w}}, \boldsymbol{t}')^{-\frac{\theta_s}{1-\rho}} \right)^{1-\rho} \right]. \tag{E.20}$$

Equivalently, define the price residual

$$F_P(\hat{\boldsymbol{p}}, \hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv \hat{\boldsymbol{p}} - \mathcal{P}(\hat{\boldsymbol{p}}, \hat{\boldsymbol{w}}, \boldsymbol{t}'). \tag{E.21}$$

The price solution is therefore the vector $\hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$ satisfying $F_P(\hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}'), \hat{\boldsymbol{w}}, \boldsymbol{t}') = 0$.

Proposition E.3 (Price indices) *Suppose Assumption 1 holds. For every $\hat{\boldsymbol{w}} \in \mathbb{R}^N_{++}$ and $\boldsymbol{t}' \in \mathbf{T}$, the map $\mathcal{P}(\cdot, \hat{\boldsymbol{w}}, \boldsymbol{t}')$ is a contraction under the sup norm with modulus at most $1 - \bar{\alpha}$. It therefore has a unique fixed point $\hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$, which yields the unique positive price-change vector $\hat{\boldsymbol{P}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$ satisfying (E.2) and (E.5). The residual F_P and the solution mappings $\hat{\boldsymbol{p}}$ and $\hat{\boldsymbol{P}}$ are continuously differentiable in wages and in tariffs in the sense of Definition 1, and $F_{P,\hat{\boldsymbol{p}}}$ is invertible at the solution.*
The price solution is also componentwise nondecreasing in wages and tariffs. In particular, for $\hat{\boldsymbol{w}} \leq \tilde{\hat{\boldsymbol{w}}}$ and $\boldsymbol{t}' \leq \tilde{\boldsymbol{t}}'$, componentwise,

$$\hat{\boldsymbol{P}}(\hat{\boldsymbol{w}}, \boldsymbol{t}') \leq \hat{\boldsymbol{P}}(\tilde{\hat{\boldsymbol{w}}}, \tilde{\boldsymbol{t}}').$$

Write $\lambda'_{mn,s} \equiv \lambda_{mn,s}\hat{\lambda}_{mn,s}$ for the counterfactual bilateral trade share implied by the price solution. At the solution, the residual Jacobians have entries

$$\begin{aligned}
\left(F_{P,\hat{\boldsymbol{p}}}\right)_{((n,s);(m,k))} &= \mathbf{1}\{(n,s) = (m,k)\} - \lambda'_{mn,s}\alpha_{m,ks}, \\
\left(F_{P,\hat{\boldsymbol{w}}}\right)_{((n,s);m)} &= -\lambda'_{mn,s}\frac{\alpha_{m,\ell s}}{\hat{w}_m}, \\
\left(F_{P,\boldsymbol{t}'}\right)_{((n,s);(m,j,k))} &= -\frac{\lambda'_{mn,s}}{1 + t'_{mn,s}}\mathbf{1}\{(j,k) = (n,s)\}, \qquad m \neq j.
\end{aligned} \tag{E.22}$$

Proof. For two candidate log-price vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, equation (E.2) implies

$$|\ln \hat{c}_{ln,s}(\boldsymbol{p}) - \ln \hat{c}_{ln,s}(\boldsymbol{q})| \leq \left(\sum_k \alpha_{l,ks}\right) \|\boldsymbol{p} - \boldsymbol{q}\|_\infty \leq (1 - \bar{\alpha})\|\boldsymbol{p} - \boldsymbol{q}\|_\infty.$$

For fixed (i, n, s), define the log of the inner aggregator by

$$G_{in,s}(\boldsymbol{p}) \equiv \ln\left[\sum_l \frac{\lambda_{iln,s}}{\Lambda_{in,s}} \exp\left(-\frac{\theta_s}{1-\rho}\ln \hat{c}_{ln,s}(\boldsymbol{p})\right)\right].$$

Because the weights $\lambda_{iln,s}/\Lambda_{in,s}$ sum to one over l, Lemma 1 and the preceding cost bound imply

$$|G_{in,s}(\boldsymbol{p}) - G_{in,s}(\boldsymbol{q})| \leq \frac{\theta_s}{1-\rho}(1 - \bar{\alpha})\|\boldsymbol{p} - \boldsymbol{q}\|_\infty.$$

Next, define the log of the outer aggregator by

$$H_{n,s}(\boldsymbol{p}) \equiv \ln\left[\sum_i \Lambda_{in,s} \exp((1-\rho)G_{in,s}(\boldsymbol{p}))\right].$$

The weights $\Lambda_{in,s}$ sum to one over i. A second application of Lemma 1 therefore gives

$$|H_{n,s}(\boldsymbol{p}) - H_{n,s}(\boldsymbol{q})| \leq \theta_s(1 - \bar{\alpha})\|\boldsymbol{p} - \boldsymbol{q}\|_\infty.$$

Since $\mathcal{P}_{n,s} = -(1/\theta_s)H_{n,s}$, it follows component by component that

$$\left|\mathcal{P}_{n,s}(\boldsymbol{p}, \hat{\boldsymbol{w}}, \boldsymbol{t}') - \mathcal{P}_{n,s}(\boldsymbol{q}, \hat{\boldsymbol{w}}, \boldsymbol{t}')\right| \leq (1 - \bar{\alpha})\|\boldsymbol{p} - \boldsymbol{q}\|_\infty.$$

Taking the maximum over (n, s) yields

$$\|\mathcal{P}(\boldsymbol{p}, \hat{\boldsymbol{w}}, \boldsymbol{t}') - \mathcal{P}(\boldsymbol{q}, \hat{\boldsymbol{w}}, \boldsymbol{t}')\|_\infty \leq (1 - \bar{\alpha})\|\boldsymbol{p} - \boldsymbol{q}\|_\infty.$$

Thus $\mathcal{P}$ is a contraction under the sup norm with modulus at most $1 - \bar{\alpha} < 1$. The Banach fixed-point theorem gives a unique $\hat{\boldsymbol{p}} \in \mathbb{R}^{NS}$; exponentiating componentwise gives the unique $\hat{\boldsymbol{P}} \gg 0$.

To establish monotonicity, first note from (E.20) and (E.2) that $\mathcal{P}$ is componentwise nondecreasing jointly in log prices, wages, and tariffs. Let $(\hat{\boldsymbol{w}}, \boldsymbol{t}') \leq (\tilde{\hat{\boldsymbol{w}}}, \tilde{\boldsymbol{t}}')$ componentwise, and set

$$\boldsymbol{p}^0 \equiv \hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}'), \qquad \boldsymbol{p}^{r+1} \equiv \mathcal{P}(\boldsymbol{p}^r, \tilde{\hat{\boldsymbol{w}}}, \tilde{\boldsymbol{t}}').$$

Order preservation gives $\boldsymbol{p}^1 \geq \boldsymbol{p}^0$ and, by induction, $\boldsymbol{p}^{r+1} \geq \boldsymbol{p}^r$ for every r. Because $\mathcal{P}(\cdot, \tilde{\hat{\boldsymbol{w}}}, \tilde{\boldsymbol{t}}')$ is a contraction, this sequence converges to its unique fixed point $\hat{\boldsymbol{p}}(\tilde{\hat{\boldsymbol{w}}}, \tilde{\boldsymbol{t}}')$. Hence

$$\hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}') \leq \hat{\boldsymbol{p}}(\tilde{\hat{\boldsymbol{w}}}, \tilde{\boldsymbol{t}}'),$$

and componentwise exponentiation proves the stated monotonicity of $\hat{\boldsymbol{P}}$.

At the fixed point, the price differential derived in (H.6) is

$$d\mathcal{P}_{n,s} = \sum_m \lambda'_{mn,s}\, d\ln \hat{c}_{mn,s}.$$

Combining this expression with (E.2) and using $F_P = \hat{\boldsymbol{p}} - \mathcal{P}$ gives (E.22). By Lemma 2, $\|\mathcal{P}_{\hat{\boldsymbol{p}}}\|_\infty \leq 1 - \bar{\alpha} < 1$, so $\mathrm{spr}(\mathcal{P}_{\hat{\boldsymbol{p}}}) < 1$. Lemma 3 then implies that $F_{P,\hat{\boldsymbol{p}}} = \mathbf{I}_{NS} - \mathcal{P}_{\hat{\boldsymbol{p}}}$ is invertible.

Finally, all arguments of the powers and logarithms in (E.20) are strictly positive. Thus F_P is continuously differentiable in $(\hat{\boldsymbol{p}}, \hat{\boldsymbol{w}})$ and in tariffs, with the boundary convention in Definition 1. The implicit function theorem gives the same regularity for $\hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$, and componentwise exponentiation gives it for $\hat{\boldsymbol{P}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$.

Q.E.D.

All residual Jacobians below are evaluated at the price solution. The implicit function theorem gives

$$\hat{\boldsymbol{p}}_{\hat{\boldsymbol{w}}} = -F_{P,\hat{\boldsymbol{p}}}^{-1} F_{P,\hat{\boldsymbol{w}}}, \qquad \hat{\boldsymbol{p}}_{\boldsymbol{t}'} = -F_{P,\hat{\boldsymbol{p}}}^{-1} F_{P,\boldsymbol{t}'}. \tag{E.23}$$

The inverse $F_{P,\hat{\boldsymbol{p}}}^{-1}$ is the propagation matrix Ψ in Section 3.2, evaluated at the counterfactual expenditure shares.

E.3.2 Trade Shares

Given $(\hat{\boldsymbol{w}}, \hat{\boldsymbol{p}}, \boldsymbol{t}')$, equation (E.3) determines the trilateral share changes, and equation (E.4) aggregates them into bilateral share changes $\hat{\boldsymbol{\lambda}}$. Write the associated counterfactual shares as

$$\lambda'_{iln,s} \equiv \lambda_{iln,s}\hat{\lambda}_{iln,s}, \qquad \lambda'_{ln,s} \equiv \sum_i \lambda'_{iln,s} = \lambda_{ln,s}\hat{\lambda}_{ln,s}, \qquad \Lambda'_{in,s} \equiv \sum_l \lambda'_{iln,s}.$$

For origins l and l', let $\eta^{ll'}_{n,s}$ be the bilateral-share elasticity introduced in Section 3.2, evaluated at the counterfactual shares:

$$\eta^{ll'}_{n,s} \equiv \frac{\partial \ln \hat{\lambda}_{ln,s}}{\partial \ln \hat{c}_{l'n,s}}.$$

Because the baseline is fixed, this is also $\partial \ln \lambda'_{ln,s}/\partial \ln \hat{c}_{l'n,s}$. Appendix H shows that

$$\eta^{ll'}_{n,s} = \frac{\theta_s}{1-\rho}\left[-\mathbf{1}\{l'=l\} + \lambda'_{l'n,s} - \rho\left(\lambda'_{l'n,s} - \sum_i \frac{\lambda'_{iln,s}}{\lambda'_{ln,s}}\frac{\lambda'_{il'n,s}}{\Lambda'_{in,s}}\right)\right].$$

In particular, the share multiplying the common destination-sector term is $\lambda'_{l'n,s}$, because l' indexes the cost being changed.

Using (E.2), the partial derivatives that hold the other two arguments fixed are

$$\begin{aligned}
\left(\frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{w}}}\right)_{((l,n,s);m)} &= \frac{\alpha_{m,\ell s}}{\hat{w}_m}\hat{\lambda}_{ln,s}\eta^{lm}_{n,s}, \\
\left(\frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{p}}}\right)_{((l,n,s);(m,k))} &= \alpha_{m,ks}\hat{\lambda}_{ln,s}\eta^{lm}_{n,s}, \\
\left(\frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \boldsymbol{t}'}\right)_{((l,n,s);(m,n',k))} &= \frac{\hat{\lambda}_{ln,s}\eta^{lm}_{n,s}}{1+t'_{mn,s}}\mathbf{1}\{(n',k)=(n,s)\}, \qquad m \neq n'.
\end{aligned} \tag{E.24}$$

Proposition E.4 (Trade shares) *Suppose Assumption 1 holds. Conditional on the price solution in Proposition E.3, equations* (E.3) *and* (E.4) *uniquely determine strictly positive trilateral and bilateral trade-share changes. The composite mapping*

$$(\hat{\boldsymbol{w}}, \boldsymbol{t}') \longmapsto \hat{\boldsymbol{\lambda}}(\hat{\boldsymbol{w}}, \hat{\boldsymbol{p}}(\hat{\boldsymbol{w}}, \boldsymbol{t}'), \boldsymbol{t}')$$

is continuously differentiable in wages and in tariffs in the sense of Definition 1.

Proof. Equation (E.2) gives a unique positive delivered-cost change for every positive wage and price vector and every admissible tariff vector. The right-hand sides of (E.3) and (E.4) then use only finite sums, products, and powers of positive objects, so they define unique positive shares. These operations are continuously differentiable, and their denominators remain positive on $\mathbf{T}$. Combining this observation with the price regularity in Proposition E.3 proves the result, including the one-sided interpretation at

tariff bounds.

Q.E.D.

Finally, the total responses of bilateral shares account for the induced price response. By the chain rule,

$$\hat{\boldsymbol{\lambda}}_{\hat{\boldsymbol{w}}} = \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{w}}} + \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{p}}} \hat{\boldsymbol{p}}_{\hat{\boldsymbol{w}}}, \qquad \hat{\boldsymbol{\lambda}}_{\boldsymbol{t}'} = \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \boldsymbol{t}'} + \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{p}}} \hat{\boldsymbol{p}}_{\boldsymbol{t}'}. \tag{E.25}$$

E.3.3 Gross Output

Equation (E.6) uses labor-market clearing to replace labor income by producer revenues. Relative to the conditional level equations in Section 3.1, this changes how output is evaluated away from equilibrium; both representations impose the same equilibrium conditions.

Given $(\hat{\boldsymbol{\lambda}}, \boldsymbol{t}')$, define $\mathbf{M} \in \mathbb{R}^{NS \times NS}$ by

$$M_{(l,s),(n,k)} \equiv \tilde{\alpha}'_{ln,sk} + \frac{\kappa'_{ln,s}}{\zeta'_n}(\xi'_{n,k} + \alpha_{n,\ell k}), \tag{E.26}$$

where the accounting objects are defined in (E.9). Equation (E.6) is then $\boldsymbol{Y}' = \mathbf{M}\boldsymbol{Y}'$.

This system determines gross output only up to scale because the columns of $\mathbf{M}$ sum to one. Specifically, one market-clearing equation is redundant. Define the stacked labor-share vector $\boldsymbol{\alpha} \in \mathbb{R}^{NS}$ by $(\boldsymbol{\alpha})_{(n,s)} \equiv \alpha_{n,\ell s}$. The wage normalization (E.8) can then be expressed as

$$\boldsymbol{\alpha}^\top \boldsymbol{Y}' = \sum_{n,s} \alpha_{n,\ell s} Y'_{n,s} = 1. \tag{E.27}$$

This is the normalization imposed when solving for $\boldsymbol{Y}'$.

Fix any row $\bar{r} \in \{1, \ldots, NS\}$, and let $\mathbf{R}^Y_{\bar{r}} \in \mathbb{R}^{(NS-1)\times NS}$ be the identity matrix with row $\bar{r}$ removed. Replacing that redundant market-clearing row by the normalization gives the square system

$$\mathbf{F}_Y(\mathbf{M})\boldsymbol{Y}' = \begin{bmatrix} \mathbf{0}_{NS-1} \\ 1 \end{bmatrix}, \qquad \mathbf{F}_Y(\mathbf{M}) \equiv \begin{bmatrix} \mathbf{R}^Y_{\bar{r}}(\mathbf{I}_{NS} - \mathbf{M}) \\ \boldsymbol{\alpha}^\top \end{bmatrix}. \tag{E.28}$$

Proposition E.5 (Gross output) *Suppose Assumption 1 holds. Conditional on the trade-share solution in Proposition E.4, the matrix $\mathbf{M}$ is strictly positive and column-stochastic. The coefficient matrix $\mathbf{F}_Y(\mathbf{M})$ in (E.28) is nonsingular, so the system has a unique solution $\boldsymbol{Y}' \gg \mathbf{0}$. The mapping $(\hat{\boldsymbol{w}}, \boldsymbol{t}') \mapsto \boldsymbol{Y}'$ is continuously differentiable in wages and in tariffs in the sense of Definition 1.*

Proof. Lemma 6 shows that $\mathbf{M} \gg \mathbf{0}$, $\mathbf{1}^\top \mathbf{M} = \mathbf{1}^\top$, and $\mathrm{spr}(\mathbf{M}) = 1$. The Perron–Frobenius theorem therefore gives a strictly positive eigenvector $\boldsymbol{v}$ satisfying $\mathbf{M}\boldsymbol{v} = \boldsymbol{v}$, and this eigenvector is unique up to scale. Because $\boldsymbol{\alpha}^\top \boldsymbol{v} > 0$, the unique normalized solution is

$$\boldsymbol{Y}' = \frac{\boldsymbol{v}}{\boldsymbol{\alpha}^\top \boldsymbol{v}} \gg \mathbf{0}.$$

It remains to verify that the square representation is nonsingular. Suppose $\mathbf{F}_Y(\mathbf{M})\boldsymbol{h} = \mathbf{0}$. Then $\boldsymbol{\alpha}^\top \boldsymbol{h} = 0$, and $(\mathbf{I}_{NS} - \mathbf{M})\boldsymbol{h}$ can be nonzero only in row $\bar{r}$. Its components nevertheless sum to zero because $\mathbf{1}^\top(\mathbf{I}_{NS} - \mathbf{M}) = \mathbf{0}^\top$. The omitted component must therefore also be zero, so $\boldsymbol{h}$ lies in the one-dimensional eigenspace spanned by $\boldsymbol{v}$. The equality $\boldsymbol{\alpha}^\top \boldsymbol{h} = 0$, together with $\boldsymbol{\alpha}^\top \boldsymbol{v} > 0$, then implies $\boldsymbol{h} = \mathbf{0}$. Hence $\mathbf{F}_Y(\mathbf{M})$ is nonsingular.

Finally, ζ_n' is bounded away from zero by Lemma 4; hence every entry of $\mathbf{M}$ is continuously differentiable in $(\hat{\boldsymbol{\lambda}}, \boldsymbol{t}')$. Proposition E.4 gives the same regularity for $\hat{\boldsymbol{\lambda}}(\hat{\boldsymbol{w}}, \boldsymbol{t}')$. Since matrix inversion is continuously differentiable on the set of nonsingular matrices,

$$\boldsymbol{Y}' = \mathbf{F}_Y(\mathbf{M})^{-1} \begin{bmatrix} \mathbf{0}_{NS-1} \\ 1 \end{bmatrix}$$

has the stated regularity.

Q.E.D.

For the derivative, hold $\boldsymbol{Y}'$ fixed and define the product map

$$\mathcal{M}(\hat{\boldsymbol{\lambda}}, \boldsymbol{t}') \equiv \mathbf{M}(\hat{\boldsymbol{\lambda}}, \boldsymbol{t}')\boldsymbol{Y}'.$$

Define

$$\boldsymbol{m}_{\hat{\boldsymbol{w}}} \equiv \mathcal{M}_{\hat{\boldsymbol{\lambda}}}\hat{\boldsymbol{\lambda}}_{\hat{\boldsymbol{w}}}, \qquad \boldsymbol{m}_{\boldsymbol{t}'} \equiv \mathcal{M}_{\hat{\boldsymbol{\lambda}}}\hat{\boldsymbol{\lambda}}_{\boldsymbol{t}'} + \mathcal{M}_{\boldsymbol{t}'}. \tag{E.29}$$

The differentials defining $\mathcal{M}_{\hat{\boldsymbol{\lambda}}}$ and $\mathcal{M}_{\boldsymbol{t}'}$ are reported in Appendix H. Differentiating (E.28) gives, for $f \in \{\hat{\boldsymbol{w}}, \boldsymbol{t}'\}$,

$$\boldsymbol{Y}_f' = \mathbf{F}_Y(\mathbf{M})^{-1} \begin{bmatrix} \mathbf{R}_{\bar{r}}^Y \boldsymbol{m}_f \\ \mathbf{0} \end{bmatrix}. \tag{E.30}$$

E.3.4 Gross Expenditure

For each country n, let $\mathbf{S}_n \in \mathbb{R}^{S \times NS}$ be the selector satisfying $\mathbf{S}_n \boldsymbol{x} = (x_{n,1}, \ldots, x_{n,S})^\top$ for every stacked country-sector vector $\boldsymbol{x}$. Write $\boldsymbol{Y}_n' \equiv \mathbf{S}_n \boldsymbol{Y}'$ and $\boldsymbol{X}_n' \equiv \mathbf{S}_n \boldsymbol{X}'$. Define

$$\begin{array}{rclcrcl} \boldsymbol{\mu}_n & \equiv & (\mu_{n,1}, \ldots, \mu_{n,S})^\top, & & \boldsymbol{\alpha}_{n,\ell} & \equiv & (\alpha_{n,\ell 1}, \ldots, \alpha_{n,\ell S})^\top, \\ \boldsymbol{\vartheta}_n' & \equiv & (\vartheta_{n,1}', \ldots, \vartheta_{n,S}')^\top, & & (\mathbf{A}_n)_{s,k} & \equiv & \alpha_{n,sk}. \end{array} \tag{E.31}$$

Equation (E.11) becomes the country-specific system

$$(\mathbf{I}_S - \boldsymbol{\mu}_n \boldsymbol{\vartheta}_n'^\top)\boldsymbol{X}_n' = \left(\mathbf{A}_n + \boldsymbol{\mu}_n \boldsymbol{\alpha}_{n,\ell}^\top\right) \boldsymbol{Y}_n'. \tag{E.32}$$

Proposition E.6 (Gross expenditure) *Suppose Assumption 1 holds. Conditional on the gross-output solution in Proposition E.5, equation* (E.32) *has a unique solution $\boldsymbol{X}_n' \gg \mathbf{0}$ for every country n. The stacked mapping $(\hat{\boldsymbol{w}}, \boldsymbol{t}') \mapsto \boldsymbol{X}'$ is continuously differentiable in wages and in tariffs in the sense of Definition 1.*

Proof. Lemma 4 gives $\zeta'_n = 1 - \boldsymbol{\vartheta}'_n{}^{\top} \boldsymbol{\mu}_n > 0$. The Sherman–Morrison formula therefore yields

$$\begin{aligned}(\mathbf{I}_S - \boldsymbol{\mu}_n \boldsymbol{\vartheta}'_n{}^{\top})^{-1} &= \mathbf{I}_S + \frac{\boldsymbol{\mu}_n \boldsymbol{\vartheta}'_n{}^{\top}}{\zeta'_n}, \\ \boldsymbol{X}'_n &= (\mathbf{I}_S - \boldsymbol{\mu}_n \boldsymbol{\vartheta}'_n{}^{\top})^{-1} \left(\mathbf{A}_n + \boldsymbol{\mu}_n \boldsymbol{\alpha}_{n,\ell}{}^{\top}\right) \boldsymbol{Y}'_n.\end{aligned} \tag{E.33}$$

This proves uniqueness. Moreover, $\left(\mathbf{A}_n + \boldsymbol{\mu}_n \boldsymbol{\alpha}_{n,\ell}{}^{\top}\right) \boldsymbol{Y}'_n \gg \mathbf{0}$ because $\boldsymbol{\mu}_n \gg \mathbf{0}$, $\boldsymbol{\alpha}_{n,\ell} \gg \mathbf{0}$, and $\boldsymbol{Y}'_n \gg \mathbf{0}$. The inverse in (E.33) is elementwise at least $\mathbf{I}_S$, so $\boldsymbol{X}'_n \gg \mathbf{0}$. Finally, Proposition E.5 and the formulas above express $\boldsymbol{X}'_n$ as a composition of continuously differentiable objects with ζ'_n bounded away from zero. This proves the stated regularity, including the one-sided interpretation at tariff bounds.

Q.E.D.

The derivative is most transparent from the linear system. For $f \in \{\hat{\boldsymbol{w}}, \boldsymbol{t}'\}$, differentiating (E.32) and solving for $\boldsymbol{X}'_{n,f}$ gives

$$\boldsymbol{X}'_{n,f} = (\mathbf{I}_S - \boldsymbol{\mu}_n \boldsymbol{\vartheta}'_n{}^{\top})^{-1} \left[\left(\mathbf{A}_n + \boldsymbol{\mu}_n \boldsymbol{\alpha}_{n,\ell}{}^{\top}\right) \mathbf{S}_n \boldsymbol{Y}'_f + \boldsymbol{\mu}_n (\boldsymbol{X}'_n)^{\top} \boldsymbol{\vartheta}'_{n,f}\right]. \tag{E.34}$$

The required accounting sensitivities follow from

$$\boldsymbol{\vartheta}'_{n,\hat{\boldsymbol{w}}} = \boldsymbol{\vartheta}'_{n,\hat{\boldsymbol{\lambda}}} \hat{\boldsymbol{\lambda}}_{\hat{\boldsymbol{w}}}, \qquad \boldsymbol{\vartheta}'_{n,\boldsymbol{t}'} = \boldsymbol{\vartheta}'_{n,\hat{\boldsymbol{\lambda}}} \hat{\boldsymbol{\lambda}}_{\boldsymbol{t}'} + \boldsymbol{\vartheta}'_{n,\boldsymbol{t}'}\big|_{\hat{\boldsymbol{\lambda}}}, \tag{E.35}$$

where the final term holds trade shares fixed. Appendix H gives the corresponding differential explicitly.

E.3.5 Tariff Revenue Shares

The counterfactual tariff-revenue share is defined in (E.13). To shorten the derivative expressions, denote its denominator by

$$\pi'_{n,\mathrm{den}} \equiv \sum_s \alpha_{n,\ell s} Y'_{n,s} + \sum_s \vartheta'_{n,s} X'_{n,s}. \tag{E.36}$$

Treating $(\hat{\boldsymbol{\lambda}}, \boldsymbol{Y}', \boldsymbol{X}', \boldsymbol{t}')$ as separate arguments, the quotient rule and (H.9) give

$$\begin{aligned} d\pi'_n = \frac{1 - \pi'_n}{\pi'_{n,\mathrm{den}}} &\times \left[\sum_{l,s} \frac{t'_{ln,s}}{1 + t'_{ln,s}} \lambda_{ln,s} X'_{n,s}\, d\hat{\lambda}_{ln,s} + \sum_s \vartheta'_{n,s}\, dX'_{n,s} \right. \\ &\left. + \sum_{l,s} \frac{\lambda'_{ln,s} X'_{n,s}}{(1 + t'_{ln,s})^2}\, dt'_{ln,s}\right] \\ &- \frac{\pi'_n}{\pi'_{n,\mathrm{den}}} \sum_s \alpha_{n,\ell s}\, dY'_{n,s}. \end{aligned} \tag{E.37}$$

Holding the other arguments fixed, the corresponding partial derivatives are

$$\begin{aligned}
\frac{\partial \pi'_n}{\partial \hat{\lambda}_{l'n',s'}} &= \frac{1-\pi'_n}{\pi'_{n,\text{den}}} \frac{t'_{l'n',s'}}{1+t'_{l'n',s'}} \lambda_{l'n',s'} X'_{n',s'} \mathbf{1}\{n'=n\}, \\
\frac{\partial \pi'_n}{\partial Y'_{n',s'}} &= -\frac{\pi'_n}{\pi'_{n,\text{den}}} \alpha_{n',\ell s'} \mathbf{1}\{n'=n\}, \\
\frac{\partial \pi'_n}{\partial X'_{n',s'}} &= \frac{1-\pi'_n}{\pi'_{n,\text{den}}} \vartheta'_{n',s'} \mathbf{1}\{n'=n\}, \\
\frac{\partial \pi'_n}{\partial t'_{l'n',s'}} &= \frac{1-\pi'_n}{\pi'_{n,\text{den}}} \frac{\lambda'_{l'n',s'} X'_{n',s'}}{(1+t'_{l'n',s'})^2} \mathbf{1}\{n'=n\}, \qquad l' \neq n'.
\end{aligned} \tag{E.38}$$

These are the direct partial derivatives. The total response of π'_n to wages or tariffs additionally includes the induced responses of $\hat{\boldsymbol{\lambda}}$, $\boldsymbol{Y}'$, and $\boldsymbol{X}'$ through the chain rule.

E.4 Derivative of Equilibrium Wages

Let $(\hat{\boldsymbol{w}}, \boldsymbol{t}')$ be a selected normalized equilibrium. Fix $\bar{n} \in \{1, \ldots, N\}$, and let $\mathbf{R}^W_{\bar{n}} \in \mathbb{R}^{(N-1)\times N}$ denote the identity matrix with row $\bar{n}$ removed. To apply the implicit function theorem, extend the formula for $\hat{\boldsymbol{Z}}$ in (E.15), and hence the scaled and projected residual $\tilde{\boldsymbol{Z}} = \Pi_b \mathbf{D}_Y \hat{\boldsymbol{Z}}$ in (E.18), to all strictly positive wage vectors in a neighborhood of the selected equilibrium, while retaining the gross-output normalization in (E.27). After substituting the conditional price, trade-share, and gross-output solutions, define the square wage residual by

$$F_W(\hat{\boldsymbol{w}}, \boldsymbol{t}') \equiv \begin{bmatrix} \mathbf{R}^W_{\bar{n}} \tilde{\boldsymbol{Z}}(\hat{\boldsymbol{w}}, \boldsymbol{t}') \\ \boldsymbol{b}^\top \hat{\boldsymbol{w}} - 1 \end{bmatrix}. \tag{E.39}$$

The first $N-1$ rows impose all but one projected residual, and the final row imposes the wage normalization. The proof of Proposition E.1 establishes that, on Δ_b, $\tilde{\boldsymbol{Z}} = \mathbf{0}$ if and only if $\hat{\boldsymbol{Z}} = \mathbf{0}$. Moreover, $\boldsymbol{b}^\top \tilde{\boldsymbol{Z}} = 0$ by construction. Since $b_{\bar{n}} > 0$, the omitted projected residual is zero whenever the retained $N-1$ projected residuals are zero. Hence the zeros of (E.39) are exactly the normalized equilibrium wages.

Let $\hat{\boldsymbol{Z}}_{\hat{\boldsymbol{w}}}$ and $\hat{\boldsymbol{Z}}_{\boldsymbol{t}'}$ denote the derivatives of the residual, including the responses of all conditional equilibrium blocks. Their rows are

$$\begin{aligned}
\left(\hat{\boldsymbol{Z}}_{\hat{\boldsymbol{w}}}\right)_{(n;\cdot)} &= \sum_s \alpha_{n,\ell s} \left(\boldsymbol{Y}'_{\hat{\boldsymbol{w}}}\right)_{((n,s);\cdot)} - b_n e_n{}^\top, \\
\left(\hat{\boldsymbol{Z}}_{\boldsymbol{t}'}\right)_{(n;\cdot)} &= \sum_s \alpha_{n,\ell s} \left(\boldsymbol{Y}'_{\boldsymbol{t}'}\right)_{((n,s);\cdot)}.
\end{aligned} \tag{E.40}$$

Because Π_b and $\mathbf{D}_Y$ contain only baseline objects, they are fixed when tariffs and counterfactual wages vary. Differentiating (E.39) therefore gives

$$F_{W,\hat{\boldsymbol{w}}} = \begin{bmatrix} \mathbf{R}^W_{\bar{n}} \Pi_b \mathbf{D}_Y \hat{\boldsymbol{Z}}_{\hat{\boldsymbol{w}}} \\ \boldsymbol{b}^\top \end{bmatrix}, \qquad F_{W,\boldsymbol{t}'} = \begin{bmatrix} \mathbf{R}^W_{\bar{n}} \Pi_b \mathbf{D}_Y \hat{\boldsymbol{Z}}_{\boldsymbol{t}'} \\ \mathbf{0}_{1\times NNS} \end{bmatrix}. \tag{E.41}$$

Assumption 2 (Local wage regularity) *At the selected normalized equilibrium, the square Jacobian $F_{W,\hat{\boldsymbol{w}}}$ in (E.41) is nonsingular.*

Proposition E.7 (Equilibrium wage derivative) *Suppose Assumptions 1 and 2 hold. The selected normalized equilibrium wage vector is locally unique and continuously differentiable in the non-domestic tariffs in the sense of Definition 1. Its derivative is*

$$\hat{\boldsymbol{w}}_{\boldsymbol{t}'} = -F_{W,\hat{\boldsymbol{w}}}^{-1} F_{W,\boldsymbol{t}'}. \tag{E.42}$$

Proof. Propositions E.3, E.4, and E.5 imply that the conditional equilibrium objects, and hence F_W, are continuously differentiable. The equivalence established above shows that the zeros of (E.39) are exactly the normalized equilibrium wages. Assumption 2 and the implicit function theorem therefore give a locally unique, continuously differentiable wage selection. Differentiating $F_W(\hat{\boldsymbol{w}}(\boldsymbol{t}'), \boldsymbol{t}') = \mathbf{0}$ gives

$$F_{W,\hat{\boldsymbol{w}}}\, d\hat{\boldsymbol{w}} + F_{W,\boldsymbol{t}'}\, d\boldsymbol{t}' = \mathbf{0}.$$

Solving this system for $d\hat{\boldsymbol{w}}$ yields (E.42). At a tariff bound, the same formula denotes the continuous feasible one-sided derivative specified in Definition 1.

Q.E.D.

E.5 Application of the Adjoint Method

The adjoint calculation applies the transpose systems associated with the equilibrium blocks in reverse order. The welfare coefficients are first propagated through gross expenditure, gross output, and trade shares. The price response to wages then determines the coefficient entering the wage adjoint, whose propagation through gross output and trade shares generates an additional price coefficient. Since both price coefficients enter the same linear price system, they can be combined before a single price–tariff pullback. This procedure yields the welfare derivative without constructing the full equilibrium Jacobians with respect to tariffs. Throughout this section, barred objects denote row vectors, and all derivatives are evaluated at the selected counterfactual equilibrium.

The welfare formula (E.12) gives

$$d\log \hat{U}_h = \frac{1}{\hat{w}_h}\, d\hat{w}_h - \sum_s \mu_{h,s}\, d\hat{p}_{h,s} + \frac{1}{1-\pi'_h}\, d\pi'_h. \tag{E.43}$$

Stack the direct partial derivatives in (E.38) into row vectors. Define the initial coefficients by

$$\begin{aligned}
\bar{\boldsymbol{w}}^{(0)} &\equiv \hat{w}_h^{-1} e_h{}^{\top}, & \bar{\boldsymbol{t}}^{(0)} &\equiv (1-\pi'_h)^{-1}\pi'_{h,\boldsymbol{t}'},\\
\bar{\boldsymbol{p}}^{(0)} &\equiv -\sum_s \mu_{h,s} e_{(h,s)}{}^{\top}, & \bar{\boldsymbol{\lambda}}^{(0)} &\equiv (1-\pi'_h)^{-1}\pi'_{h,\hat{\boldsymbol{\lambda}}},\\
\bar{\boldsymbol{Y}}^{(0)} &\equiv (1-\pi'_h)^{-1}\pi'_{h,\boldsymbol{Y}'}, & \bar{\boldsymbol{X}}^{(0)} &\equiv (1-\pi'_h)^{-1}\pi'_{h,\boldsymbol{X}'}.
\end{aligned} \tag{E.44}$$

Thus (E.43) can be written as

$$\begin{aligned}
d\log \hat{U}_h = \bar{\boldsymbol{w}}^{(0)} d\hat{\boldsymbol{w}} + \bar{\boldsymbol{p}}^{(0)} d\hat{\boldsymbol{p}} + \bar{\boldsymbol{\lambda}}^{(0)} d\hat{\boldsymbol{\lambda}} + \bar{\boldsymbol{Y}}^{(0)} d\boldsymbol{Y}'\\
+ \bar{\boldsymbol{X}}^{(0)} d\boldsymbol{X}' + \bar{\boldsymbol{t}}^{(0)} d\boldsymbol{t}'.
\end{aligned} \tag{E.45}$$

Gross expenditure. For each country n, denote the country block of the expenditure coefficient by $\bar{\boldsymbol{X}}_n^{(0)} \equiv \bar{\boldsymbol{X}}^{(0)} \mathbf{S}_n{}^\top$. Let $\boldsymbol{q}_{X,n} \in \mathbb{R}^S$ solve

$$\left(\mathbf{I}_S - \boldsymbol{\mu}_n (\boldsymbol{\vartheta}'_n)^\top\right)^\top \boldsymbol{q}_{X,n} = (\bar{\boldsymbol{X}}_n^{(0)})^\top. \tag{E.46}$$

Multiplying the differential of (E.32) by $\boldsymbol{q}_{X,n}{}^\top$ gives

$$\begin{aligned} \bar{\boldsymbol{X}}_n^{(0)} d\boldsymbol{X}'_n = \boldsymbol{q}_{X,n}{}^\top \left(\mathbf{A}_n + \boldsymbol{\mu}_n \boldsymbol{\alpha}_{n,\ell}{}^\top\right) \mathbf{S}_n d\boldsymbol{Y}' \\ + (\boldsymbol{q}_{X,n}{}^\top \boldsymbol{\mu}_n)(\boldsymbol{X}'_n)^\top d\boldsymbol{\vartheta}'_n. \end{aligned} \tag{E.47}$$

The first term contributes to the coefficient on gross output. Expanding the second term using (H.9) gives the corresponding contributions to the coefficients on trade shares and tariffs. Define

$$\begin{aligned} \bar{\boldsymbol{Y}}^{(1)} &\equiv \bar{\boldsymbol{Y}}^{(0)} + \sum_n \boldsymbol{q}_{X,n}{}^\top \left(\mathbf{A}_n + \boldsymbol{\mu}_n \boldsymbol{\alpha}_{n,\ell}{}^\top\right) \mathbf{S}_n, \\ \bar{\lambda}_{ln,s}^{(1)} &\equiv \bar{\lambda}_{ln,s}^{(0)} + (\boldsymbol{q}_{X,n}{}^\top \boldsymbol{\mu}_n) X'_{n,s} \frac{t'_{ln,s}}{1 + t'_{ln,s}} \lambda_{ln,s}, \\ \bar{t}_{ln,s}^{(1)} &\equiv \bar{t}_{ln,s}^{(0)} + (\boldsymbol{q}_{X,n}{}^\top \boldsymbol{\mu}_n) X'_{n,s} \frac{\lambda'_{ln,s}}{(1 + t'_{ln,s})^2}. \end{aligned} \tag{E.48}$$

Substitution of (E.47) into (E.45) eliminates $d\boldsymbol{X}'$ and gives

$$\begin{aligned} d \log \hat{U}_h = \bar{\boldsymbol{w}}^{(0)} d\hat{\boldsymbol{w}} + \bar{\boldsymbol{p}}^{(0)} d\hat{\boldsymbol{p}} + \bar{\boldsymbol{\lambda}}^{(1)} d\hat{\boldsymbol{\lambda}} \\ + \bar{\boldsymbol{Y}}^{(1)} d\boldsymbol{Y}' + \bar{\boldsymbol{t}}^{(1)} d\boldsymbol{t}'. \end{aligned} \tag{E.49}$$

Gross output. Let $\boldsymbol{q}_Y \in \mathbb{R}^{NS}$ solve the transpose of the gross-output system,

$$\mathbf{F}_Y(\mathbf{M})^\top \boldsymbol{q}_Y = (\bar{\boldsymbol{Y}}^{(1)})^\top. \tag{E.50}$$

Equation (E.30) then implies

$$\begin{aligned} \bar{\boldsymbol{Y}}^{(1)} d\boldsymbol{Y}' = \boldsymbol{q}_Y{}^\top \begin{bmatrix} \mathbf{R}_{\bar{r}}^Y \mathcal{M}_{\hat{\boldsymbol{\lambda}}} \\ \mathbf{0}_{1 \times NNS} \end{bmatrix} d\hat{\boldsymbol{\lambda}} \\ + \boldsymbol{q}_Y{}^\top \begin{bmatrix} \mathbf{R}_{\bar{r}}^Y \mathcal{M}_{\boldsymbol{t}'} \\ \mathbf{0}_{1 \times NNS} \end{bmatrix} d\boldsymbol{t}'. \end{aligned} \tag{E.51}$$

Accordingly, define

$$\begin{aligned} \bar{\boldsymbol{\lambda}}^{(2)} &\equiv \bar{\boldsymbol{\lambda}}^{(1)} + \boldsymbol{q}_Y{}^\top \begin{bmatrix} \mathbf{R}_{\bar{r}}^Y \mathcal{M}_{\hat{\boldsymbol{\lambda}}} \\ \mathbf{0}_{1 \times NNS} \end{bmatrix}, \\ \bar{\boldsymbol{t}}^{(2)} &\equiv \bar{\boldsymbol{t}}^{(1)} + \boldsymbol{q}_Y{}^\top \begin{bmatrix} \mathbf{R}_{\bar{r}}^Y \mathcal{M}_{\boldsymbol{t}'} \\ \mathbf{0}_{1 \times NNS} \end{bmatrix}. \end{aligned} \tag{E.52}$$

The resulting differential is

$$d \log \hat{U}_h = \bar{\boldsymbol{w}}^{(0)} d\hat{\boldsymbol{w}} + \bar{\boldsymbol{p}}^{(0)} d\hat{\boldsymbol{p}} + \bar{\boldsymbol{\lambda}}^{(2)} d\hat{\boldsymbol{\lambda}} + \bar{\boldsymbol{t}}^{(2)} d\boldsymbol{t}'. \tag{E.53}$$

The product-map derivatives appearing in these expressions are given in Appendix H.

Trade shares. Substitution of the direct partial derivatives in (E.24) into (E.53) eliminates $d\hat{\boldsymbol{\lambda}}$. The resulting coefficient rows, together with their component representations, are

$$\begin{aligned}
\bar{\boldsymbol{w}}^{\lambda} &\equiv \bar{\boldsymbol{\lambda}}^{(2)} \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{w}}}, \qquad \bar{w}^{\lambda}_{l'} = \sum_{l,n,s} \bar{\lambda}^{(2)}_{ln,s} \frac{\alpha_{l',\ell s}}{\hat{w}_{l'}} \hat{\lambda}_{ln,s} \eta^{ll'}_{n,s}, \\
\bar{\boldsymbol{p}}^{\lambda} &\equiv \bar{\boldsymbol{\lambda}}^{(2)} \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{p}}}, \qquad \bar{p}^{\lambda}_{l',s'} = \sum_{l,n,s} \bar{\lambda}^{(2)}_{ln,s} \alpha_{l',s's} \hat{\lambda}_{ln,s} \eta^{ll'}_{n,s}, \\
\bar{\boldsymbol{t}}^{\lambda} &\equiv \bar{\boldsymbol{\lambda}}^{(2)} \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \boldsymbol{t}'}, \quad \bar{t}^{\lambda}_{l'n',s'} = \sum_{l} \bar{\lambda}^{(2)}_{ln',s'} \frac{\hat{\lambda}_{ln',s'} \eta^{ll'}_{n',s'}}{1 + t'_{l'n',s'}}, \quad l' \neq n'.
\end{aligned} \tag{E.54}$$

Collecting these contributions, define

$$\begin{aligned}
\bar{\boldsymbol{w}}^{(1)} &\equiv \bar{\boldsymbol{w}}^{(0)} + \bar{\boldsymbol{w}}^{\lambda}, & \bar{\boldsymbol{p}}^{(1)} &\equiv \bar{\boldsymbol{p}}^{(0)} + \bar{\boldsymbol{p}}^{\lambda}, \\
\bar{\boldsymbol{t}}^{(3)} &\equiv \bar{\boldsymbol{t}}^{(2)} + \bar{\boldsymbol{t}}^{\lambda}.
\end{aligned} \tag{E.55}$$

Hence

$$d \log \hat{U}_h = \bar{\boldsymbol{w}}^{(1)} d\hat{\boldsymbol{w}} + \bar{\boldsymbol{p}}^{(1)} d\hat{\boldsymbol{p}} + \bar{\boldsymbol{t}}^{(3)} d\boldsymbol{t}'. \tag{E.56}$$

Price response to wages. At fixed tariffs, the first identity in (E.23) gives the price response $d\hat{\boldsymbol{p}} = \hat{\boldsymbol{p}}_{\hat{\boldsymbol{w}}} d\hat{\boldsymbol{w}}$. The resulting wage coefficient is

$$\bar{\boldsymbol{w}}^{(2)} \equiv \bar{\boldsymbol{w}}^{(1)} + \bar{\boldsymbol{p}}^{(1)} \hat{\boldsymbol{p}}_{\hat{\boldsymbol{w}}}. \tag{E.57}$$

Substituting the wage-induced price response into (E.56) gives

$$d \log \hat{U}_h = \bar{\boldsymbol{w}}^{(2)} d\hat{\boldsymbol{w}} + \left(\bar{\boldsymbol{t}}^{(3)} + \bar{\boldsymbol{p}}^{(1)} \hat{\boldsymbol{p}}_{\boldsymbol{t}'} \right) d\boldsymbol{t}'. \tag{E.58}$$

The tariff-induced price response is retained for combination with the price coefficient generated by the wage block below.

Wages. Let $\boldsymbol{q}_W \in \mathbb{R}^N$ solve

$$F_{W,\hat{\boldsymbol{w}}}{}^{\top} \boldsymbol{q}_W = (\bar{\boldsymbol{w}}^{(2)})^{\top}, \tag{E.59}$$

and partition it as $\boldsymbol{q}_W = ((\boldsymbol{q}_W^Z)^{\top}, q_W^b)^{\top}$. Here $\boldsymbol{q}_W^Z \in \mathbb{R}^{N-1}$ multiplies the retained labor-market residuals, while q_W^b multiplies the wage normalization. Equations (E.41), (E.42), and (E.59) imply

$$\begin{aligned}
\bar{\boldsymbol{w}}^{(2)} \hat{\boldsymbol{w}}_{\boldsymbol{t}'} &= -\boldsymbol{q}_W{}^{\top} F_{W,\boldsymbol{t}'} \\
&= -(\boldsymbol{q}_W^Z)^{\top} \mathbf{R}^W_{\bar{n}} \Pi_b \mathbf{D}_Y \hat{\boldsymbol{Z}}_{\boldsymbol{t}'}.
\end{aligned} \tag{E.60}$$

The second equality follows because the normalization row has no direct tariff derivative.

By (E.40), the gross-output coefficient induced by (E.60) is

$$\bar{Y}^W_{n,s} \equiv -\frac{\alpha_{n,\ell s}}{\sum_k Y_{n,k}} \left[\Pi_b(\mathbf{R}^W_{\bar{n}})^\top \boldsymbol{q}^Z_W\right]_n. \tag{E.61}$$

It follows that

$$-\boldsymbol{q}_W{}^\top F_{W,\boldsymbol{t}'} = \bar{\boldsymbol{Y}}^W \boldsymbol{Y}'_{\boldsymbol{t}'}. \tag{E.62}$$

Let $\boldsymbol{q}^W_Y \in \mathbb{R}^{NS}$ solve

$$\mathbf{F}_Y(\mathbf{M})^\top \boldsymbol{q}^W_Y = (\bar{\boldsymbol{Y}}^W)^\top. \tag{E.63}$$

Propagating this coefficient through gross output gives

$$\begin{aligned} \bar{\boldsymbol{\lambda}}^W &\equiv (\boldsymbol{q}^W_Y)^\top \begin{bmatrix} \mathbf{R}^Y_{\bar{r}} \mathcal{M}_{\hat{\boldsymbol{\lambda}}} \\ \mathbf{0}_{1\times NNS} \end{bmatrix}, \\ \bar{\boldsymbol{t}}^{W,Y} &\equiv (\boldsymbol{q}^W_Y)^\top \begin{bmatrix} \mathbf{R}^Y_{\bar{r}} \mathcal{M}_{\boldsymbol{t}'} \\ \mathbf{0}_{1\times NNS} \end{bmatrix}, \\ \bar{\boldsymbol{Y}}^W \boldsymbol{Y}'_{\boldsymbol{t}'} &= \bar{\boldsymbol{\lambda}}^W \hat{\boldsymbol{\lambda}}_{\boldsymbol{t}'} + \bar{\boldsymbol{t}}^{W,Y}. \end{aligned} \tag{E.64}$$

Propagating the remaining coefficient through trade shares gives

$$\begin{aligned} \bar{\boldsymbol{p}}^W &\equiv \bar{\boldsymbol{\lambda}}^W \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \hat{\boldsymbol{p}}}, \quad \bar{p}^W_{l',s'} = \sum_{l,n,s} \bar{\lambda}^W_{ln,s} \alpha_{l',s's} \hat{\lambda}_{ln,s} \eta^{ll'}_{n,s}, \\ \bar{\boldsymbol{t}}^{W,\lambda} &\equiv \bar{\boldsymbol{\lambda}}^W \frac{\partial \hat{\boldsymbol{\lambda}}}{\partial \boldsymbol{t}'}, \quad \bar{t}^{W,\lambda}_{l'n',s'} = \sum_{l} \bar{\lambda}^W_{ln',s'} \frac{\hat{\lambda}_{ln',s'} \eta^{ll'}_{n',s'}}{1 + t'_{l'n',s'}}, \quad l' \neq n'. \end{aligned} \tag{E.65}$$

The total share derivative in (E.25) yields

$$\bar{\boldsymbol{\lambda}}^W \hat{\boldsymbol{\lambda}}_{\boldsymbol{t}'} = \bar{\boldsymbol{p}}^W \hat{\boldsymbol{p}}_{\boldsymbol{t}'} + \bar{\boldsymbol{t}}^{W,\lambda}. \tag{E.66}$$

Combining (E.60), (E.64), and (E.66) gives the wage-induced tariff coefficient:

$$\bar{\boldsymbol{w}}^{(2)} \hat{\boldsymbol{w}}_{\boldsymbol{t}'} = \bar{\boldsymbol{t}}^{W,Y} + \bar{\boldsymbol{t}}^{W,\lambda} + \bar{\boldsymbol{p}}^W \hat{\boldsymbol{p}}_{\boldsymbol{t}'}. \tag{E.67}$$

Combined price pullback. The price coefficient $\bar{\boldsymbol{p}}^{(1)}$ from (E.56) and the wage-induced coefficient $\bar{\boldsymbol{p}}^W$ in (E.65) multiply the same tariff-induced price response. By linearity, they can be added before solving the price transpose system. Define

$$\bar{\boldsymbol{p}}^{(2)} \equiv \bar{\boldsymbol{p}}^{(1)} + \bar{\boldsymbol{p}}^W, \tag{E.68}$$

and let $\boldsymbol{q}_P \in \mathbb{R}^{NS}$ solve

$$F_{P,\hat{\boldsymbol{p}}}{}^\top \boldsymbol{q}_P = (\bar{\boldsymbol{p}}^{(2)})^\top. \tag{E.69}$$

The combined price–tariff coefficient is

$$\bar{\boldsymbol{t}}^P \equiv -\boldsymbol{q}_P{}^\top F_{P,\boldsymbol{t}'}. \tag{E.70}$$

Using the explicit derivative in (E.22), its components are

$$\bar{t}^{P}_{l'n',s'} = q_{P,(n',s')} \frac{\lambda'_{l'n',s'}}{1 + t'_{l'n',s'}}, \qquad l' \neq n'. \tag{E.71}$$

This coefficient collects both $\bar{\boldsymbol{p}}^{(1)}\hat{\boldsymbol{p}}_{\boldsymbol{t}'}$ and $\bar{\boldsymbol{p}}^{W}\hat{\boldsymbol{p}}_{\boldsymbol{t}'}$.

Combining the direct coefficient in (E.56), the two non-price coefficients generated by the wage block, and the combined price coefficient yields the final welfare differential

$$d \log \hat{U}_h = \left(\bar{\boldsymbol{t}}^{(3)} + \bar{\boldsymbol{t}}^{W,Y} + \bar{\boldsymbol{t}}^{W,\lambda} + \bar{\boldsymbol{t}}^{P} \right) d\boldsymbol{t}'. \tag{E.72}$$

The home-country welfare gradient therefore has components

$$\left(\nabla_{\boldsymbol{t}_h} \log \hat{U}_h \right)_{(l,s)} = \bar{t}^{(3)}_{lh,s} + \bar{t}^{W,Y}_{lh,s} + \bar{t}^{W,\lambda}_{lh,s} + \bar{t}^{P}_{lh,s}, \qquad l \neq h. \tag{E.73}$$

Since baseline welfare is invariant to $\boldsymbol{t}'$, this gradient also equals $\nabla_{\boldsymbol{t}_h} \log U'_h$. Given the price–wage sensitivities in (E.23) and the wage Jacobian in (E.41), the reverse pass requires one additional price transpose solve and does not construct the price–tariff, output–tariff, expenditure–tariff, or wage–tariff Jacobians.

F Lemmas and Theorems

Lemma 1 (A log-sum-exp Lipschitz bound) *Let $n \in \mathbb{N}$ and let $\boldsymbol{a} = (a_1, \ldots, a_n)$ satisfy $a_j \geq 0$ and $\sum_{j=1}^n a_j = 1$. Define*

$$\mathrm{LSE}_{\boldsymbol{a}}(\boldsymbol{x}) \equiv \ln\left(\sum_{j=1}^n a_j e^{x_j}\right), \qquad \boldsymbol{x} \in \mathbb{R}^n .$$

Then $\mathrm{LSE}_{\boldsymbol{a}}$ is 1-Lipschitz with respect to the sup norm:

$$|\mathrm{LSE}_{\boldsymbol{a}}(\boldsymbol{x}) - \mathrm{LSE}_{\boldsymbol{a}}(\boldsymbol{y})| \leq \|\boldsymbol{x} - \boldsymbol{y}\|_\infty \qquad \textit{for all } \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n .$$

Proof. Let $\delta \equiv \|\boldsymbol{x} - \boldsymbol{y}\|_\infty$. Then $x_j \leq y_j + \delta$ for every j, so

$$\sum_{j=1}^n a_j e^{x_j} \leq e^\delta \sum_{j=1}^n a_j e^{y_j}.$$

Taking logs gives $\mathrm{LSE}_{\boldsymbol{a}}(\boldsymbol{x}) \leq \mathrm{LSE}_{\boldsymbol{a}}(\boldsymbol{y}) + \delta$. Reversing the roles of $\boldsymbol{x}$ and $\boldsymbol{y}$ gives the opposite inequality and proves the result.

Q.E.D.

Lemma 2 (Derivative bound from a global Lipschitz constant) *Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be differentiable and globally Lipschitz under $\|\cdot\|_\infty$ with constant $L \geq 0$:*

$$\|f(\boldsymbol{x}) - f(\boldsymbol{y})\|_\infty \leq L\|\boldsymbol{x} - \boldsymbol{y}\|_\infty \qquad \textit{for all } \boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n .$$

Then the induced operator norm of the Jacobian satisfies

$$\|Df(\boldsymbol{x})\|_\infty \leq L \qquad \textit{for all } \boldsymbol{x} \in \mathbb{R}^n .$$

Proof. Fix $\boldsymbol{x} \in \mathbb{R}^n$ and a nonzero direction $\boldsymbol{h} \in \mathbb{R}^n$. For $t \neq 0$, Lipschitz continuity implies

$$\left\| \frac{f(\boldsymbol{x} + t\boldsymbol{h}) - f(\boldsymbol{x})}{t} \right\|_\infty \leq L\|\boldsymbol{h}\|_\infty.$$

Letting $t \to 0$ yields $\|Df(\boldsymbol{x})\boldsymbol{h}\|_\infty \leq L\|\boldsymbol{h}\|_\infty$. Taking the supremum over $\|\boldsymbol{h}\|_\infty = 1$ proves the claim.

Q.E.D.

Lemma 3 (Neumann series and invertibility) *Let $A \in \mathbb{R}^{n \times n}$ have spectral radius $\mathrm{spr}(A) < 1$. Then $I_n - A$ is invertible and*

$$(I_n - A)^{-1} = \sum_{k=0}^\infty A^k.$$

The series converges absolutely in any submultiplicative matrix norm. Consequently, if $A \geq 0$ elementwise, then $(I_n - A)^{-1} \geq 0$ elementwise.

Proof. By Gelfand's formula,

$$\operatorname{spr}(A) = \lim_{k\to\infty} \|A^k\|^{1/k}.$$

Choose $c \in (\operatorname{spr}(A), 1)$. For all sufficiently large k, $\|A^k\| < c^k$, so $\sum_{k=0}^{\infty} A^k$ converges absolutely. Define $S_N \equiv \sum_{k=0}^{N} A^k$. Then

$$(I_n - A)S_N = S_N(I_n - A) = I_n - A^{N+1}.$$

Because $A^{N+1} \to 0$, taking limits proves the inverse formula. If A is elementwise nonnegative, every term in the series is elementwise nonnegative, which proves the final claim.

Q.E.D.

Lemma 4 (Tariff accounting) *Suppose Assumption 1 holds, and fix $\boldsymbol{t}' \in \mathbf{T}$. The objects in (E.9) satisfy*

$$\begin{aligned} 0 \le \vartheta'_{n,s} &< \frac{\bar{t}}{1+\bar{t}} < \frac{1}{2}, & \zeta'_n &> \frac{1}{1+\bar{t}} > \frac{1}{2}, \\ \xi'_{n,k} &\ge 0, & \tilde{\alpha}'_{ln,sk} &\ge 0, \\ \kappa'_{ln,s} &> 0. \end{aligned}$$

and

$$\begin{aligned} \sum_l \frac{\lambda'_{ln,s}}{1+t'_{ln,s}} &= 1 - \vartheta'_{n,s}, & \sum_{l,s} \kappa'_{ln,s} &= \zeta'_n, \\ \sum_{l,s} \tilde{\alpha}'_{ln,sk} &= 1 - \alpha_{n,\ell k} - \xi'_{n,k}. \end{aligned}$$

Proof. The first bound follows because $\vartheta'_{n,s}$ is the $\lambda'_{ln,s}$-weighted average of $t'_{ln,s}/(1+t'_{ln,s})$, whose largest possible value is $\bar{t}/(1+\bar{t})$; the inequality is strict because the domestic wedge is zero and $\lambda'_{nn,s} > 0$. Averaging across sectors with weights $\mu_{n,s}$ gives the bound on ζ'_n. The sign restrictions follow immediately from the definitions. Finally,

$$\frac{1}{1+t} = 1 - \frac{t}{1+t}$$

and $\sum_l \lambda'_{ln,s} = 1$ give the first identity. Summing the definitions of κ' and $\tilde{\alpha}'$ and using $\alpha_{n,\ell k} + \sum_s \alpha_{n,sk} = 1$ gives the remaining two identities.

Q.E.D.

Lemma 5 (Boundary behavior of labor excess demand) *Suppose Assumption 1 holds. Fix $\boldsymbol{t}' \in \mathbf{T}$. Let $\{\hat{\boldsymbol{w}}^q\}_{q=1}^{\infty} \subset \Delta_b$ converge to a nonzero vector $\hat{\boldsymbol{w}}^0 \in \mathbb{R}_+^N$ with $\hat{w}_n^0 = 0$ for at least one country n. Then*

$$\max_n \frac{\hat{Z}_n(\hat{\boldsymbol{w}}^q, \boldsymbol{t}')}{\hat{w}_n^q} \longrightarrow +\infty.$$

Proof. This is the multi-sector counterpart of the boundary argument in Alvarez and Lucas (2007, Appendix B). Let $\delta_q \equiv \min_n \hat{w}_n^q$. Because the limit lies on the boundary, $\delta_q \to 0$. Let $\underline{b} \equiv \min_n b_n > 0$. The normalization $\boldsymbol{b}^\top \hat{\boldsymbol{w}}^q = 1$ implies

$$\delta_q \mathbf{1} \le \hat{\boldsymbol{w}}^q \le \underline{b}^{-1}\mathbf{1}. \tag{F.1}$$

Proposition E.3 establishes that the price solution is componentwise nondecreasing in wages. The price system is also homogeneous of degree one jointly in wages and prices. Define $c_P \equiv \min_{n,s} \hat{P}_{n,s}(\mathbf{1}, \boldsymbol{t}') > 0$. The lower bound in (F.1) therefore implies

$$\hat{P}_{n,s}(\hat{\boldsymbol{w}}^q, \boldsymbol{t}') \geq \hat{P}_{n,s}(\delta_q \mathbf{1}, \boldsymbol{t}') = \delta_q \hat{P}_{n,s}(\mathbf{1}, \boldsymbol{t}') \geq c_P \delta_q \qquad \text{for every } (n,s). \tag{F.2}$$

An upper bound on prices in each producing country is also needed. Strict positivity of the baseline domestic shares implies that, for every (l,s), retaining a domestic production term in (E.5) gives a finite constant $\chi^P_{l,s} > 0$, independent of q, such that

$$\hat{P}_{l,s}(\hat{\boldsymbol{w}}^q, \boldsymbol{t}') \leq \chi^P_{l,s} (\hat{w}^q_l)^{\alpha_{l,\ell s}} \prod_k \hat{P}_{l,k}(\hat{\boldsymbol{w}}^q, \boldsymbol{t}')^{\alpha_{l,ks}}. \tag{F.3}$$

Let $(\mathbf{A}^P_l)_{s,k} \equiv \alpha_{l,ks}$, and let $\hat{\boldsymbol{p}}^q_l$ collect the log price changes $\log \hat{P}_{l,s}(\hat{\boldsymbol{w}}^q, \boldsymbol{t}')$ across sectors in country l. Also define $\boldsymbol{\chi}^P_l \equiv (\log \chi^P_{l,1}, \ldots, \log \chi^P_{l,S})^\top$. Taking logs of (F.3), subtracting $\log \hat{w}^q_l$, and using $\alpha_{l,\ell s} + \sum_k \alpha_{l,ks} = 1$ gives

$$(\mathbf{I}_S - \mathbf{A}^P_l)\left(\hat{\boldsymbol{p}}^q_l - \log \hat{w}^q_l \mathbf{1}_S\right) \leq \boldsymbol{\chi}^P_l.$$

Every row sum of $\mathbf{A}^P_l$ is at most $1 - \bar{\alpha} < 1$. Lemma 3 therefore implies $(\mathbf{I}_S - \mathbf{A}^P_l)^{-1} \geq 0$. Define the finite constant

$$C_P \equiv \max_{l,s} \exp\left(\left[(\mathbf{I}_S - \mathbf{A}^P_l)^{-1} \boldsymbol{\chi}^P_l\right]_s\right) > 0.$$

Multiplying the preceding inequality by the nonnegative inverse and exponentiating yields

$$\hat{P}_{l,s}(\hat{\boldsymbol{w}}^q, \boldsymbol{t}') \leq C_P \hat{w}^q_l \qquad \text{for every } (l,s). \tag{F.4}$$

Choose k_q such that $\hat{w}^q_{k_q} = \delta_q$. Equations (F.2)–(F.4), the cost equation (E.2), and constant returns to scale imply constants $0 < c_c < C_c < \infty$ such that

$$\hat{c}_{k_q n,s} \leq C_c \delta_q, \qquad \hat{c}_{mn,s} \geq c_c \delta_q \quad \text{for every } (m,n,s,q).$$

Thus the delivered cost of the lowest-wage country is at most a fixed multiple of every competing cost. Conditional on any technology source i, (E.3) implies

$$\frac{\lambda'_{ik_q n,s}}{\Lambda'_{in,s}} = \frac{(\lambda_{ik_q n,s}/\Lambda_{in,s})\hat{c}^{-\theta_s/(1-\rho)}_{k_q n,s}}{\sum_m (\lambda_{imn,s}/\Lambda_{in,s})\hat{c}^{-\theta_s/(1-\rho)}_{mn,s}} \geq \frac{\lambda_{ik_q n,s}}{\Lambda_{in,s}} \left(\frac{c_c}{C_c}\right)^{\theta_s/(1-\rho)}.$$

The finitely many baseline conditional shares are strictly positive. Taking the minimum of the right side over all source, production-country, destination, and sector indices, and then averaging across sources with weights $\Lambda'_{in,s}$, gives a constant $c_\lambda > 0$ such that

$$\lambda'_{k_q n,s}(\hat{\boldsymbol{w}}^q, \boldsymbol{t}') \geq c_\lambda \qquad \text{for every } (n,s,q). \tag{F.5}$$

Let $\mathcal{L}_n(\hat{\boldsymbol{w}}^q) \equiv \sum_s \alpha_{n,\ell s} Y'_{n,s}(\hat{\boldsymbol{w}}^q)$. By (E.27), $\sum_n \mathcal{L}_n = 1$, so for each q there is a country j_q with $\mathcal{L}_{j_q} \geq$

$1/N$. Let $\underline{\mu} \equiv \min_{n,r} \mu_{n,r} > 0$. Equation (E.11) and nonnegativity of all its terms imply

$$X'_{j_q,s} \geq \mu_{j_q,s} \mathcal{L}_{j_q} \geq \frac{\underline{\mu}}{N} \qquad \text{for every } s.$$

Define $c_Y \equiv c_\lambda \underline{\mu}/[N(1+\bar{t})] > 0$. Using the counterfactual version of goods-market clearing (16), (F.5), and $t'_{ln,s} \leq \bar{t}$, the following bound is obtained for any fixed sector s:

$$Y'_{k_q,s} \geq \frac{\lambda'_{k_q j_q,s}}{1 + t'_{k_q j_q,s}} X'_{j_q,s} \geq c_Y.$$

Since $\alpha_{k_q,\ell s} \geq \bar{\alpha}$, $\mathcal{L}_{k_q} \geq \bar{\alpha} c_Y$. Hence

$$\max_n \frac{\hat{Z}_n(\hat{\boldsymbol{w}}^q, \boldsymbol{t}')}{\hat{w}_n^q} \geq \frac{\mathcal{L}_{k_q}(\hat{\boldsymbol{w}}^q)}{\hat{w}_{k_q}^q} - \max_n b_n \geq \frac{\bar{\alpha} c_Y}{\delta_q} - \max_n b_n \longrightarrow +\infty.$$

Q.E.D.

Lemma 6 (Output matrix) *Under Assumption 1, the matrix* $\mathbf{M}$ *in* (E.26) *is strictly positive and column-stochastic. It is therefore primitive, with* $\mathrm{spr}(\mathbf{M}) = 1$.

Proof. Lemma 4 and Assumption 1 imply

$$M_{(l,s),(n,k)} \geq \frac{\kappa'_{ln,s}}{\zeta'_n} \alpha_{n,\ell k} > 0.$$

For each column (n,k), the identities in Lemma 4 give

$$\begin{aligned} \sum_{l,s} M_{(l,s),(n,k)} &= (1 - \alpha_{n,\ell k} - \xi'_{n,k}) + \frac{\xi'_{n,k} + \alpha_{n,\ell k}}{\zeta'_n} \sum_{l,s} \kappa'_{ln,s} \\ &= 1. \end{aligned}$$

Thus $\mathbf{M}$ is strictly positive and column-stochastic. Strict positivity implies primitivity. Moreover, $\mathbf{1}^\top \mathbf{M} = \mathbf{1}^\top$, so 1 is an eigenvalue, while $\|\mathbf{M}\|_1 = 1$ implies $\mathrm{spr}(\mathbf{M}) \leq 1$. Hence $\mathrm{spr}(\mathbf{M}) = 1$.

Q.E.D.

G Exact-Hat Algebra

The normalized cost terms and the relationships between technology-source and production-location shares are given by (A.2), (A.5), and (A.10) in Appendix A.

Recall that for any variable x, $\hat{x} = x'/x$ where x is the value under baseline tariffs $t_{ln,s}$ and x' is the value under the counterfactual tariffs $t'_{ln,s}$.

Tariff-inclusive cost (A.15) **in exact hats.** The tariff-inclusive unit cost in exact changes is

$$\hat{c}_{iln,s} = \frac{\left(1 + t'_{ln,s}\right)}{(1 + t_{ln,s})} \times \frac{\tilde{\gamma}_{il,s}\tau_{ln,s}}{\tilde{\gamma}_{il,s}\tau_{ln,s}} \frac{(w'_l)^{\alpha_{l,\ell s}} \prod_k (P'_{l,k})^{\alpha_{l,ks}}}{w_l^{\alpha_{l,\ell s}} \prod_k P_{l,k}^{\alpha_{l,ks}}},$$

$$\hat{c}_{iln,s} = \frac{\left(1 + t'_{ln,s}\right)}{(1 + t_{ln,s})} \hat{w}_l^{\alpha_{l,\ell s}} \prod_k \hat{P}_{l,k}^{\alpha_{l,ks}}.$$

Because the exact-hat cost change is independent of technology source i, that index is omitted:

$$\hat{c}_{iln,s} = \hat{c}_{ln,s} \quad \forall i = 1, \ldots, N.$$

This is the cost equation in (E.2) of Appendix E.

Trilateral shares (A.16) **in exact hats.** Taking exact changes of the trilateral trade share yields

$$\hat{\lambda}_{iln,s} = \frac{\hat{D}_{iln,s}\hat{D}_{in,s}^{-\rho}}{\hat{D}_{n,s}}. \tag{G.1}$$

The system is therefore solved for $\hat{D}_{iln,s}$, $\hat{D}_{in,s}$, and $\hat{D}_{n,s}$ in that order, and the results are substituted into (G.1).

Taking exact changes of $D_{iln,s}$ yields

$$\hat{D}_{iln,s} = \hat{c}_{ln,s}^{-\frac{\theta_s}{1-\rho}}. \tag{G.2}$$

For $D_{in,s}$,

$$\hat{D}_{in,s} = \frac{\sum_{l'} D'_{il'n,s}}{D_{in,s}} = \sum_{l'} \frac{D_{il'n,s}}{D_{in,s}} \hat{D}_{il'n,s}.$$

Using the first identity in (A.10),

$$\hat{D}_{in,s} = \sum_{l} \frac{\lambda_{iln,s}}{\Lambda_{in,s}} \hat{D}_{iln,s}. \tag{G.3}$$

Taking exact changes of $D_{n,s}$ yields

$$\hat{D}_{n,s} = \frac{\sum_{i'}(D'_{i'n,s})^{1-\rho}}{\sum_i (D_{in,s})^{1-\rho}} = \sum_{i'}\left(\frac{(D'_{i'n,s})^{1-\rho}}{D_{n,s}} \times \frac{(D_{i'n,s})^{1-\rho}}{(D_{i'n,s})^{1-\rho}}\right) = \sum_{i'}\frac{D^{1-\rho}_{i'n,s}}{D_{n,s}}\hat{D}^{1-\rho}_{i'n,s}$$

Using the last identity in (A.10),

$$\hat{D}_{n,s} = \sum_i \Lambda_{in,s}\hat{D}^{1-\rho}_{in,s} \tag{G.4}$$

Substituting (G.2)–(G.4) into (G.1) gives (E.3):

$$\hat{\lambda}_{iln,s} = \hat{c}_{ln,s}^{-\frac{\theta_s}{1-\rho}} \frac{\left(\sum_m \frac{\lambda_{imn,s}}{\Lambda_{in,s}}\hat{c}_{mn,s}^{-\frac{\theta_s}{1-\rho}}\right)^{-\rho}}{\sum_j \Lambda_{jn,s}\left(\sum_m \frac{\lambda_{jmn,s}}{\Lambda_{jn,s}}\hat{c}_{mn,s}^{-\frac{\theta_s}{1-\rho}}\right)^{1-\rho}}.$$

Bilateral shares (A.17) **in exact hats.** Using (G.1), the exact-hat representation of the bilateral trade share is

$$\hat{\lambda}_{ln,s} \equiv \frac{\sum_i \lambda'_{iln,s}}{\lambda_{ln,s}} = \frac{\sum_i \lambda_{iln,s}\hat{\lambda}_{iln,s}}{\lambda_{ln,s}} = \frac{\sum_i \lambda_{iln,s}\hat{D}_{iln,s}\hat{D}^{-\rho}_{in,s}}{\lambda_{ln,s}\hat{D}_{n,s}}. \tag{G.5}$$

Substituting (G.2)–(G.4) into (G.5) gives the bilateral-share equation (E.4) in Appendix E.

Using (A.10) and (G.4), the exact-hat representation of the multinational-production share is

$$\hat{\Lambda}_{in,s} = \frac{\hat{D}^{1-\rho}_{in,s}}{\hat{D}_{n,s}} = \frac{\hat{D}^{1-\rho}_{in,s}}{\sum_j \Lambda_{jn,s}\hat{D}^{1-\rho}_{jn,s}}. \tag{G.6}$$

Substituting (G.2) and (G.3) into (G.6) yields $\hat{\Lambda}_{in,s}$ in terms of baseline values and $\hat{c}_{iln,s}$.

Price index (A.18) **in exact hats.** The price index in exact changes is

$$\hat{P}_{n,s} = \frac{\tilde{\Gamma}_s\left(D'_{n,s}\right)^{-\frac{1}{\theta_s}}}{\tilde{\Gamma}_s\left(D_{n,s}\right)^{-\frac{1}{\theta_s}}} = \left(\hat{D}_{n,s}\right)^{-\frac{1}{\theta_s}}. \tag{G.7}$$

Substituting (G.4) into (G.7) yields (E.5).

Labor income (A.22) **in exact hats.**

$$\frac{w'_n L_n}{w_n L_n} = \frac{\sum_s \alpha_{n,\ell s} Y'_{n,s}}{\sum_k \alpha_{n,\ell k} Y_{n,k}}$$

$$\hat{w}_n = \sum_s \frac{\alpha_{n,\ell s} Y'_{n,s}}{\sum_k \alpha_{n,\ell k} Y_{n,k}}$$

This yields (E.7) in Appendix E. Using the baseline labor-market equation in (A.22), exact changes of the normalization (A.23) give (E.8).

Counterfactual accounting objects. Using $\lambda'_{ln,s} = \lambda_{ln,s}\hat{\lambda}_{ln,s}$ in the definitions (A.25) gives the counterfactual objects in (E.9) of Appendix E. These objects enter the gross-output and gross-expenditure equations below.

Gross output (A.20) **in exact hats.** Substituting final expenditure into goods-market clearing gives

$$Y'_{l,s} = \sum_{n} \frac{\kappa'_{ln,s}}{\zeta'_n} w'_n L_n + \sum_{n,k} \left(\tilde{\alpha}'_{ln,sk} + \frac{\kappa'_{ln,s}}{\zeta'_n} \xi'_{n,k} \right) Y'_{n,k}.$$

Labor-market clearing, $w'_n L_n = \sum_k \alpha_{n,\ell k} Y'_{n,k}$, then yields (E.6) in Appendix E.

Gross expenditure (A.21) **in exact hats.** Final and intermediate demand imply

$$X'_{n,s} = \mu_{n,s} w'_n L_n + \mu_{n,s} \sum_k \vartheta'_{n,k} X'_{n,k} + \sum_k \alpha_{n,sk} Y'_{n,k}.$$

Substituting labor-market clearing gives (E.11) in Appendix E.

Aggregate prices and welfare. Taking exact changes of the aggregate price index (A.19) gives (E.10) in Appendix E. Evaluating the tariff-revenue share (A.24) at counterfactual shares and expenditures gives (E.13). Finally, the ratio of welfare across the counterfactual and baseline equilibria in (A.13) gives (E.12), which evaluates the objective in (A.14).

H Derivation of Derivatives

This appendix collects the detailed derivatives used throughout Appendix E. It first differentiates the exact-hat price and trade-share system and then the counterfactual accounting identities used by the remaining equilibrium blocks. The derivation begins with the exact-hat representations in (G.2)–(G.4). Baseline objects are fixed. Cost changes $\hat{c}_{ln,s}$ are first treated as separate arguments; wage and price responses enter through (E.2). Tariff differentials satisfy $dt'_{nn,s} = 0$, as in Definition 1.

H.1 Derivatives of $\hat{D}_{iln,s}$, $\hat{D}_{in,s}$, and $\hat{D}_{n,s}$

For a positive weighted sum $Z = \sum_j a_j x_j$ with constant weights a_j, the log differential is

$$d \ln Z = \sum_j \frac{a_j x_j}{\sum_m a_m x_m} d \ln x_j. \tag{H.1}$$

Applying this rule to (G.2) gives

$$d \ln \hat{D}_{iln,s} = -\frac{\theta_s}{1-\rho} d \ln \hat{c}_{ln,s}, \qquad \frac{\partial \ln \hat{D}_{iln,s}}{\partial \ln \hat{c}_{l'n',s'}} = -\frac{\theta_s}{1-\rho} \mathbf{1}\{(l', n', s') = (l, n, s)\}. \tag{H.2}$$

The exact-hat share identities imply

$$\frac{\hat{D}_{imn,s}}{\hat{D}_{in,s}} = \frac{\hat{\lambda}_{imn,s}}{\hat{\Lambda}_{in,s}}, \qquad \frac{\lambda_{imn,s}}{\Lambda_{in,s}} \frac{\hat{D}_{imn,s}}{\hat{D}_{in,s}} = \frac{\lambda'_{imn,s}}{\Lambda'_{in,s}}. \tag{H.3}$$

The first equality relates relative changes in the technology-source components to relative changes in their shares. Multiplying it by the baseline conditional share gives the second equality, which converts the log-sum weight below into the corresponding counterfactual conditional share. Thus,

$$d \ln \hat{D}_{in,s} = -\frac{\theta_s}{1-\rho} \sum_l \frac{\lambda'_{iln,s}}{\Lambda'_{in,s}} d \ln \hat{c}_{ln,s}, \tag{H.4}$$

and

$$\frac{\partial \ln \hat{D}_{in,s}}{\partial \ln \hat{c}_{l'n',s'}} = -\frac{\theta_s}{1-\rho} \frac{\lambda'_{il'n,s}}{\Lambda'_{in,s}} \mathbf{1}\{(n', s') = (n, s)\}.$$

Similarly, the counterfactual weight on technology source i is $\Lambda_{in,s} \hat{D}_{in,s}^{1-\rho} / \hat{D}_{n,s} = \Lambda'_{in,s}$. Hence

$$\begin{aligned} d \ln \hat{D}_{n,s} &= (1-\rho) \sum_i \Lambda'_{in,s} d \ln \hat{D}_{in,s} \\ &= -\theta_s \sum_l \lambda'_{ln,s} d \ln \hat{c}_{ln,s}, \end{aligned} \tag{H.5}$$

where $\lambda'_{ln,s} = \sum_i \lambda'_{iln,s}$. It follows that

$$\frac{\partial \ln \hat{D}_{n,s}}{\partial \ln \hat{c}_{l'n',s'}} = -\theta_s \lambda'_{l'n,s} \mathbf{1}\{(n',s') = (n,s)\}.$$

H.2 Derivative of Price Indices

The price map satisfies $\mathcal{P}_{n,s} = -(1/\theta_s) \ln \hat{D}_{n,s}$. Equation (H.5) therefore gives

$$d\mathcal{P}_{n,s} = \sum_l \lambda'_{ln,s}\, d \ln \hat{c}_{ln,s}. \tag{H.6}$$

At the fixed point, $d\hat{p}_{n,s} = d\mathcal{P}_{n,s}$. Because the baseline tariff is fixed, (E.2) implies

$$d \ln \hat{c}_{ln,s} = \frac{\alpha_{l,\ell s}}{\hat{w}_l}\, d\hat{w}_l + \sum_k \alpha_{l,ks}\, d\hat{p}_{l,k} + \frac{dt'_{ln,s}}{1 + t'_{ln,s}}.$$

Substitution into (H.6) yields the three price-residual Jacobians in (E.22).

H.3 Derivatives of Trilateral and Bilateral Trade Shares

Trilateral shares. Log-differentiating (G.1) gives

$$d \ln \hat{\lambda}_{iln,s} = -\frac{\theta_s}{1-\rho}\, d \ln \hat{c}_{ln,s} - \rho\, d \ln \hat{D}_{in,s} - d \ln \hat{D}_{n,s}.$$

Using (H.4) and (H.5),

$$d \ln \hat{\lambda}_{iln,s} = \sum_{l'} \left[-\frac{\theta_s}{1-\rho} \mathbf{1}\{l' = l\} + \rho \frac{\theta_s}{1-\rho} \frac{\lambda'_{il'n,s}}{\Lambda'_{in,s}} + \theta_s \lambda'_{l'n,s} \right] d \ln \hat{c}_{l'n,s}. \tag{H.7}$$

Thus,

$$\frac{\partial \ln \hat{\lambda}_{iln,s}}{\partial \ln \hat{c}_{l'n',s'}} = \left[-\frac{\theta_s}{1-\rho} \mathbf{1}\{l' = l\} + \rho \frac{\theta_s}{1-\rho} \frac{\lambda'_{il'n,s}}{\Lambda'_{in,s}} + \theta_s \lambda'_{l'n,s} \right] \mathbf{1}\{(n',s') = (n,s)\}.$$

Bilateral shares. Since $\hat{\lambda}_{ln,s} = \sum_i (\lambda_{iln,s}/\lambda_{ln,s}) \hat{\lambda}_{iln,s}$, the log-sum rule gives

$$d \ln \hat{\lambda}_{ln,s} = \sum_i \frac{\lambda'_{iln,s}}{\lambda'_{ln,s}}\, d \ln \hat{\lambda}_{iln,s}.$$

Combining this expression with (H.7) yields

$$\begin{aligned}\frac{\partial \ln \hat{\lambda}_{ln,s}}{\partial \ln \hat{c}_{l'n',s'}} &= \left[-\frac{\theta_s}{1-\rho}\mathbf{1}\{l'=l\} + \rho\frac{\theta_s}{1-\rho}\sum_i \frac{\lambda'_{iln,s}}{\lambda'_{ln,s}}\frac{\lambda'_{il'n,s}}{\Lambda'_{in,s}} + \theta_s\lambda'_{l'n,s}\right]\mathbf{1}\{(n',s')=(n,s)\} \\ &= \eta^{ll'}_{n,s}\mathbf{1}\{(n',s')=(n,s)\}.\end{aligned} \tag{H.8}$$

For $l, l' \neq n$, the relative-expenditure elasticity in (12) is therefore $\eta^{ll'}_{n,s} - \eta^{nl'}_{n,s}$, evaluated at the counterfactual shares, where l' indexes the origin whose cost changes.

H.4 Derivatives of Counterfactual Accounting Objects

The accounting objects in (E.9) depend on policy through the counterfactual bilateral shares and tariffs. Because the baseline shares are fixed, $d\lambda'_{ln,s} = \lambda_{ln,s}\, d\hat{\lambda}_{ln,s}$. Applying the product and quotient rules to their definitions yields

$$\begin{aligned} d\vartheta'_{n,s} &= \sum_l \left[\frac{t'_{ln,s}}{1+t'_{ln,s}}\, d\lambda'_{ln,s} + \frac{\lambda'_{ln,s}}{(1+t'_{ln,s})^2}\, dt'_{ln,s}\right], \\ d\zeta'_n &= -\sum_s \mu_{n,s}\, d\vartheta'_{n,s}, \qquad d\xi'_{n,k} = \sum_s \alpha_{n,sk}\, d\vartheta'_{n,s}, \\ d\tilde{\alpha}'_{ln,sk} &= \alpha_{n,sk}\left[\frac{d\lambda'_{ln,s}}{1+t'_{ln,s}} - \frac{\lambda'_{ln,s}}{(1+t'_{ln,s})^2}\, dt'_{ln,s}\right], \\ d\kappa'_{ln,s} &= \mu_{n,s}\left[\frac{d\lambda'_{ln,s}}{1+t'_{ln,s}} - \frac{\lambda'_{ln,s}}{(1+t'_{ln,s})^2}\, dt'_{ln,s}\right]. \end{aligned} \tag{H.9}$$

These expressions provide the Jacobians used below. For each argument, the corresponding partial derivative is obtained by collecting its differential while holding the other arguments fixed.

H.5 Derivative of Gross Output

The gross-output derivative follows directly from the product $\mathcal{M}(\hat{\boldsymbol{\lambda}}, \boldsymbol{t}') \equiv \mathbf{M}(\hat{\boldsymbol{\lambda}}, \boldsymbol{t}')\boldsymbol{Y}'$. Holding $\boldsymbol{Y}'$ fixed and differentiating the (l,s) component gives

$$\begin{aligned} d_{(\hat{\boldsymbol{\lambda}},\boldsymbol{t}')}\mathcal{M}_{(l,s)} = \sum_{n,k}\Bigg[&d\tilde{\alpha}'_{ln,sk} + \frac{d\kappa'_{ln,s}}{\zeta'_n}\left(\xi'_{n,k} + \alpha_{n,\ell k}\right) - \frac{\kappa'_{ln,s}}{(\zeta'_n)^2}\, d\zeta'_n\left(\xi'_{n,k} + \alpha_{n,\ell k}\right) \\ &+ \frac{\kappa'_{ln,s}}{\zeta'_n}\, d\xi'_{n,k}\Bigg] Y'_{n,k}. \end{aligned} \tag{H.10}$$

Substituting (H.9) and collecting coefficients gives the partial derivatives $\mathcal{M}_{\hat{\boldsymbol{\lambda}}}$ and $\mathcal{M}_{\boldsymbol{t}'}$ used in (E.29). Adding $\mathbf{M}\, d\boldsymbol{Y}'$ and differentiating the normalized square system (E.28) then yields (E.30).

H.6 Derivative of Gross Expenditure

For each country n, differentiate the expenditure system (E.32) separately from gross output:

$$(\mathbf{I}_S - \boldsymbol{\mu}_n(\boldsymbol{\vartheta}'_n)^\top)\boldsymbol{X}'_n = \left(\mathbf{A}_n + \boldsymbol{\mu}_n\boldsymbol{\alpha}_{n,\ell}{}^\top\right)\boldsymbol{Y}'_n.$$

Its differential is

$$(\mathbf{I}_S - \boldsymbol{\mu}_n(\boldsymbol{\vartheta}'_n)^\top)d\boldsymbol{X}'_n = \left(\mathbf{A}_n + \boldsymbol{\mu}_n\boldsymbol{\alpha}_{n,\ell}{}^\top\right)d\boldsymbol{Y}'_n + \boldsymbol{\mu}_n(\boldsymbol{X}'_n)^\top d\boldsymbol{\vartheta}'_n.$$

Solving this system with the inverse in (E.33) gives (E.34); hence no separate quotient-rule expansion is required.

I Computational Implementation

This appendix describes the computational steps used to solve for equilibria and optimal tariffs. Appendix E derives the equations and welfare gradient. The implementation separates the equilibrium solution at a given tariff vector from the search over tariffs. I first describe the MP model and then explain how the same routines implement the no-MP and trade-only models.

I.1 Solving for an Equilibrium

For a given tariff vector, I solve for wages using an outer fixed-point iteration, following Alvarez and Lucas (2007). I begin with an initial guess for wage hats and repeat the following steps until labor markets clear.

1. *Prices and input costs.* Given wage hats, I solve (E.5) by damped fixed-point iteration in log price hats, updating input costs using (E.2) at each step. The price iteration continues until convergence before moving to the sourcing block. Logarithms keep the implied price levels positive. The converged prices provide the initial price guess for the next wage iteration.
2. *Sourcing shares.* Given the wage and converged price hats, I compute delivered-cost changes from (E.2) and trilateral share changes from (E.3). I recover counterfactual trilateral shares as $\lambda'_{iln,s} = \lambda_{iln,s}\hat{\lambda}_{iln,s}$ and aggregate them into bilateral shares using (E.4).
3. *Gross output.* Using the new bilateral shares and tariffs, I construct the coefficients in (E.9) and solve (E.6). Gross output is obtained from the positive eigenvector of the mapping $\mathbf{M}$ in (E.26), with the normalization in (E.27). The code applies this mapping to vectors without constructing the full country–sector matrix. If the eigensolver does not return a valid positive solution, a normalized power iteration provides a fallback.
4. *Labor-market clearing.* I evaluate the labor-market residual in (E.15). If markets do not clear, I update wage hats using the scaled and projected residual in (E.18) and the normalized mapping in (E.19), then repeat the price, sourcing, and output steps. The wage-update step size increases when the residual improves and decreases otherwise.

Once wages converge, I recover gross expenditure, tariff revenue, and welfare from (E.11), (E.13), and (E.12). The equilibrium calculation requires positive wages, prices, and gross output and well-defined sourcing and accounting objects. In numerical checks using several different initial wage guesses, the algorithm appears to converge to the same equilibrium for a given tariff vector.

I.2 Solving for Optimal Tariffs

I use MATLAB's `fmincon` routine to minimize $-\ln \hat{U}_h$ over Home's tariff instruments, with each tariff constrained to lie between zero and 99%. The routine uses an interior-point method and a limited-memory BFGS approximation to the Hessian. Starting from an initial tariff vector, each objective evaluation calls the equilibrium solver described above. Once that equilibrium has converged, the analytical derivative routines return the gradient of $-\ln \hat{U}_h$ with respect to the active tariff instruments. These derivatives include the induced response of equilibrium wages and are computed using the adjoint method in Appendix E.

The optimizer uses the objective and gradient to choose a new tariff vector within the bounds. It updates its approximation to curvature using changes in tariffs and gradients across iterations, rather than calculating the full Hessian. Each new candidate requires a further equilibrium solve and gradient evaluation. This sequence continues until the optimizer's convergence criteria are met. Thus, the equilibrium iteration is nested within the tariff optimization: the former clears labor markets at a given policy, while the latter changes policy to increase Home's equilibrium welfare.

I solve the fully differentiated exporter–sector problem from five initial tariff vectors. The first sets every tariff to zero; the second and third set every tariff to 15% and 20%, respectively. The fourth and fifth are random schedules, with each tariff drawn as $0.99\sqrt{u}$ for independent $u \sim \mathcal{U}[0,1]$. These draws cover the feasible interval and place more probability on high tariffs than uniform draws. I fix the random seed and use the same five initial vectors across models. Each vector initiates a separate optimization with its own sequence of equilibrium and gradient evaluations. I accept an optimal tariff vector only if the optimizer converges and the bound-constrained first-order conditions are satisfied. Among accepted runs, I retain the tariff vector with the highest welfare. Multiple starts assess sensitivity to initialization; they do not establish global optimality.

I.3 Counterfactuals with No Multinational Production

Both benchmarks preserve the MP model's positive baseline bilateral shares, gross output, gross expenditure, and cost and final-consumption shares. To use the common solver, I represent their bilateral sourcing shares in the trilateral array as

$$\lambda_{iln,s} = \begin{cases} \lambda_{ln,s}, & i = l, \\ 0, & i \neq l. \end{cases}$$

In the no-MP model, this restriction reflects the prohibition on using technology abroad. In the trade-only model, it is a computational representation of bilateral trade: technology ownership is not a separate economic margin. In both cases, the zero entries remain zero when tariffs change.

The price and bilateral-share equations follow by evaluating the sums in (E.5) and (E.4) on this diagonal support. Each technology source has only one production location with a positive weight, so $\lambda_{iln,s}/\Lambda_{in,s} = \mathbf{1}\{i = l\}$ and $\Lambda_{in,s} = \lambda_{in,s}$. For a shape parameter θ_s^b, the resulting equations are

$$\hat{P}_{n,s} = \left(\sum_l \lambda_{ln,s} \hat{c}_{ln,s}^{-\theta_s^b} \right)^{-1/\theta_s^b},$$

$$\hat{\lambda}_{ln,s} = \frac{\hat{c}_{ln,s}^{-\theta_s^b}}{\sum_m \lambda_{mn,s} \hat{c}_{mn,s}^{-\theta_s^b}},$$

where delivered-cost changes continue to satisfy (E.2). I use $\theta_s^b = \theta_s$ for the no-MP model and $\theta_s^b = \theta_s^{\text{trade}} = \theta_s/(1-\rho)$ for the trade-only model.

Feeding the diagonal trilateral array above, including its zero off-diagonal entries, into the computational routines that solve the MP model using (E.2)–(E.13) yields the correct equilibrium equations for both counterfactuals. With the respective shape parameters, the sourcing and price calculations in (E.3)–(E.5) reduce to the bilateral equations above. The zero entries do not hinder the algorithm because the routines divide by positive aggregate shares and location sums, rather than by individual trilateral shares. For every technology source and destination–sector pair, the location sum contains a positive term at $l = i$, so its powers and normalizations remain well defined. The off-diagonal entries of the array corresponding to (E.3) are finite scaling factors multiplied by zero baseline shares. Their counterfactual flows and derivatives therefore remain zero, without evaluating a ratio of a counterfactual zero flow to a baseline zero flow.

The gradient routines also normalize only by positive location sums and bilateral shares. On the diagonal support, the conditional production-location weights are zero or one, and the bilateral-share cost elasticity reduces to

$$\frac{\partial \ln \lambda'_{ln,s}}{\partial \ln \hat{c}_{l'n,s}} = \theta_s^b (\lambda'_{l'n,s} - \mathbf{1}\{l = l'\}).$$

This is the derivative of the reduced bilateral-share equation above. The common routines therefore return the same price and sourcing derivatives as the reduced bilateral system. The remaining equilibrium blocks in (E.6)–(E.13), the wage iteration, and the adjoint calculations use these bilateral shares and derivatives. Thus, the zero off-diagonal flows do not introduce a division by zero into either the equilibrium solver or the welfare gradient.

J Multinational Production and Trade-Only Models

This appendix explains how the multinational-production and trade model in Section 1, referred to here as the MP model, nests a trade-only production structure, and what is preserved when its parameters are recalibrated.

J.1 The Trade Model

The trade model described in Section 4.4 has the same preferences, production structure, and general-equilibrium conditions as the MP model, but omits multinational-production costs and uses independent productivity draws across production locations. Denote its iceberg trade costs by $\tau^{\text{trade}}_{ln,s}$. The input cost $u^{\text{trade}}_{l,s}$ is given by (3), evaluated at the trade-only equilibrium wages and sectoral prices. Productivity draws $A^{\text{trade}}_{l,s}(\omega)$ are independent across locations, goods, and sectors. The scale $T^{\text{trade}}_{l,s} > 0$ and shape $\theta^{\text{trade}}_{s} > 0$ govern the productivity level and dispersion across goods, respectively:

$$\Pr[A^{\text{trade}}_{l,s}(\omega) \leq a] = \exp\left[-\left(\frac{a}{T^{\text{trade}}_{l,s}}\right)^{-\theta^{\text{trade}}_{s}}\right], \qquad a > 0. \tag{J.1}$$

For $\theta^{\text{trade}}_{s} > \sigma_s - 1$, the bilateral expenditure share is

$$\lambda^{\text{trade}}_{ln,s} = \frac{\left[\frac{1}{T^{\text{trade}}_{l,s}}\tau^{\text{trade}}_{ln,s}(1+t_{ln,s})u^{\text{trade}}_{l,s}\right]^{-\theta^{\text{trade}}_{s}}}{\sum_{m}\left[\frac{1}{T^{\text{trade}}_{m,s}}\tau^{\text{trade}}_{mn,s}(1+t_{mn,s})u^{\text{trade}}_{m,s}\right]^{-\theta^{\text{trade}}_{s}}}. \tag{J.2}$$

The sectoral price index is

$$P^{\text{trade}}_{n,s} = \tilde{\Gamma}^{\text{trade}}_{s}\left\{\sum_{l}\left[\frac{1}{T^{\text{trade}}_{l,s}}\tau^{\text{trade}}_{ln,s}(1+t_{ln,s})u^{\text{trade}}_{l,s}\right]^{-\theta^{\text{trade}}_{s}}\right\}^{-1/\theta^{\text{trade}}_{s}}, \tag{J.3}$$

where $\tilde{\Gamma}^{\text{trade}}_{s} \equiv \Gamma(1-(\sigma_s-1)/\theta^{\text{trade}}_{s})^{1/(1-\sigma_s)}$. Technology ownership does not enter this representation. The trade elasticity for $l, l' \neq n$ is

$$\frac{\partial \ln(\lambda^{\text{trade}}_{ln,s}/\lambda^{\text{trade}}_{nn,s})}{\partial \ln[\tau^{\text{trade}}_{l'n,s}(1+t_{l'n,s})]} = -\theta^{\text{trade}}_{s}\mathbf{1}\{l = l'\}. \tag{J.4}$$

J.2 Prohibiting Multinational Production

In the MP model, let $\gamma_{il,s} \to \infty$ for every $i \neq l$ while retaining the other primitives. Only production in a technology's source country remains feasible. Using $\gamma_{ii,s} = 1$ and the cost-adjusted technology terms

defined in (A.2) gives

$$D_{in,s} \longrightarrow D_{iin,s} = \left[\frac{1}{T_{ii,s}}\tau_{in,s}(1+t_{in,s})u_{i,s}\right]^{-\theta_s/(1-\rho)}, \quad D_{in,s}^{1-\rho} \longrightarrow \left[\frac{1}{T_{ii,s}}\tau_{in,s}(1+t_{in,s})u_{i,s}\right]^{-\theta_s}. \tag{J.5}$$

The correlation parameter therefore disappears from the aggregate trade block. Since technologies are independent across sources, the surviving productivity draws are independent across production locations. The expenditure shares become

$$\lambda_{iln,s}^{\text{nomp}} = \mathbf{1}\{i=l\}\lambda_{ln,s}^{\text{nomp}}, \qquad \lambda_{ln,s}^{\text{nomp}} = \frac{\left[\frac{1}{T_{ll,s}}\tau_{ln,s}(1+t_{ln,s})u_{l,s}\right]^{-\theta_s}}{\sum_m \left[\frac{1}{T_{mm,s}}\tau_{mn,s}(1+t_{mn,s})u_{m,s}\right]^{-\theta_s}}. \tag{J.6}$$

The corresponding price index is

$$P_{n,s}^{\text{nomp}} = \tilde{\Gamma}_s \left\{\sum_l \left[\frac{1}{T_{ll,s}}\tau_{ln,s}(1+t_{ln,s})u_{l,s}\right]^{-\theta_s}\right\}^{-1/\theta_s}. \tag{J.7}$$

With $\theta_s^{\text{trade}} = \theta_s$, $T_{l,s}^{\text{trade}} = T_{ll,s}$, and $\tau_{ln,s}^{\text{trade}} = \tau_{ln,s}$, equations (J.6) and (J.7) are identical to the trade-only equations (J.2) and (J.3). The no-MP computations use this reduced bilateral system with positive baseline bilateral shares. The off-diagonal trilateral shares are identically zero and are not assigned exact hats; the strictly positive trilateral assumptions of Appendix E do not apply to this benchmark.

J.3 Independence of Productivity across Production Locations

Setting $\rho = 0$ gives a second route to a trade-only representation. It leaves MP feasible when off-diagonal MP costs are finite. In this case, productivities are independent across production locations as well as across sources. Substituting $\rho = 0$ into (7) and summing across technology sources gives the bilateral share

$$\lambda_{ln,s} = \frac{\sum_i \left[\frac{1}{T_{il,s}}\gamma_{il,s}\tau_{ln,s}(1+t_{ln,s})u_{l,s}\right]^{-\theta_s}}{\sum_m \sum_i \left[\frac{1}{T_{im,s}}\gamma_{im,s}\tau_{mn,s}(1+t_{mn,s})u_{m,s}\right]^{-\theta_s}}. \tag{J.8}$$

Define the effective productivity scale in location l as

$$\overline{T}_{l,s} \equiv \left[\sum_i (T_{il,s}/\gamma_{il,s})^{\theta_s}\right]^{1/\theta_s}. \tag{J.9}$$

The best technology available in l, $\max_i A_{il,s}(\omega)/\gamma_{il,s}$, then has distribution

$$\Pr\left[\max_i \frac{A_{il,s}(\omega)}{\gamma_{il,s}} \le a\right] = \prod_i \exp[-(T_{il,s}/(\gamma_{il,s}a))^{\theta_s}] = \exp[-(\overline{T}_{l,s}/a)^{\theta_s}].$$

These maxima are independent across locations. The bilateral shares in (J.8) and the sectoral price indices therefore equal those in the trade model with $T_{l,s}^{\text{trade}} = \overline{T}_{l,s}$, $\theta_s^{\text{trade}} = \theta_s$, and $\tau_{ln,s}^{\text{trade}} = \tau_{ln,s}$.

Source ownership still determines trilateral flows, but adds no independent substitution term to aggregate bilateral shares. This differs from prohibiting MP: foreign technologies remain included in $\overline{T}_{l,s}$. A constant trade elasticity therefore does not imply that multinational production is absent.

J.4 Recalibration and Parameter Matching

Reproducing the MP model's bilateral shares is a separate requirement from imposing either of these structural restrictions. Let $\lambda_{ln,s}^{\text{target}}$ denote the shares to be matched at the comparison equilibrium. Dividing (J.2) by the domestic share, using $\tau_{nn,s}^{\text{trade}} = 1$ and $t_{nn,s} = 0$, gives the matching condition

$$\frac{T_{l,s}^{\text{trade}}/[\tau_{ln,s}^{\text{trade}}(1+t_{ln,s})u_{l,s}^{\text{trade}}]}{T_{n,s}^{\text{trade}}/u_{n,s}^{\text{trade}}} = \left(\frac{\lambda_{ln,s}^{\text{target}}}{\lambda_{nn,s}^{\text{target}}}\right)^{1/\theta_s^{\text{trade}}}. \tag{J.10}$$

Equation (J.10) fixes each origin's competitiveness relative to domestic production. Different combinations of technology scales and trade costs can satisfy it, so bilateral shares do not separately identify these parameters. The parameters must respect the iceberg-cost restrictions, and wages and input prices must satisfy the equilibrium conditions in Section 1. Matching shares alone does not match price levels or responses to tariff changes.

Without recalibration, prohibiting multinational production leaves the original diagonal scales $T_{ll,s}$ and trade costs to determine the new equilibrium. With recalibration, those scales and costs are adjusted to reproduce the target bilateral shares. In both cases, the shape parameter remains θ_s, and all expenditure is assigned to source–location pairs with $i = l$. A direct trade-only calibration instead allows θ_s^{trade} to be chosen separately.

Two parameter links answer different questions. The choice $\theta_s^{\text{trade}} = \theta_s$ preserves marginal productivity dispersion and corresponds to the prohibitive-cost trade structure. Alternatively,

$$\theta_s^{\text{trade}} = \frac{\theta_s}{1-\rho} \tag{J.11}$$

matches the MP model's conditional sensitivity to a production-location cost change:

$$-\frac{\partial \ln(\lambda_{iln,s}/\lambda_{inn,s})}{\partial \ln[\tau_{ln,s}(1+t_{ln,s})]} = \frac{\theta_s}{1-\rho} = -\frac{\partial \ln(\lambda_{ln,s}^{\text{trade}}/\lambda_{nn,s}^{\text{trade}})}{\partial \ln[\tau_{ln,s}^{\text{trade}}(1+t_{ln,s})]}. \tag{J.12}$$

For $l \neq n$, this equality follows from (9) and (J.4), holding wages and sectoral prices fixed. It matches the MP model's conditional elasticity to the trade model's aggregate elasticity. The MP model's aggregate elasticity also depends on technology-source composition. Either parameter link can be combined with recalibrated technology scales and trade costs to match the target shares, but the links imply different responses from that allocation.

K Cross-Nested CES Equivalence

An Armington framework treats goods produced in different locations as distinct and imperfectly substitutable products (Armington 1969). Thus, trade in an Armington framework is generated by deterministic product differentiation, rather than by the endogenous selection of the lowest-cost supplier for each good under Ricardian productivity differences. To reproduce the aggregate demand system of the present model, the Armington products must be differentiated jointly by technology source i and production location l.

Apart from the sector-specific moment constant $\tilde{\Gamma}_s$, the sectoral price index can be written as

$$\tilde{P}_{n,s} \equiv \frac{P_{n,s}}{\tilde{\Gamma}_s} = \left\{ \sum_i \left[\sum_l [\tilde{\gamma}_{il,s} \tilde{\tau}_{ln,s} (1 + t_{ln,s})]^{-\frac{\theta_s}{1-\rho}} \right]^{1-\rho} \right\}^{-1/\theta_s}. \tag{K.1}$$

The inner sum in (K.1) has the CES form for products using the same technology source but produced in different locations. Give the (i, l, s) Armington product delivered to n the deterministic price $\tilde{\gamma}_{il,s} \tilde{\tau}_{ln,s} (1 + t_{ln,s})$. If these products are combined with an elasticity of substitution $1 + \theta_s/(1-\rho)$, their quantity composite is

$$Q^A_{in,s} \equiv \left[\sum_l q_{iln,s}^{\frac{\theta_s}{\theta_s + 1 - \rho}} \right]^{\frac{\theta_s + 1 - \rho}{\theta_s}}. \tag{K.2}$$

Minimizing expenditure on the location products for a unit of $Q^A_{in,s}$ gives the technology-source price index

$$\tilde{P}_{in,s} \equiv \left[\sum_l [\tilde{\gamma}_{il,s} \tilde{\tau}_{ln,s} (1 + t_{ln,s})]^{-\frac{\theta_s}{1-\rho}} \right]^{-\frac{1-\rho}{\theta_s}}, \tag{K.3}$$

which is $D_{in,s}^{-(1-\rho)/\theta_s}$ by (A.2). The outer sum in (K.1) can then be reproduced by combining the technology-source composites with an elasticity of substitution $1 + \theta_s$:

$$Q^A_{n,s} \equiv \left[\sum_i \left(Q^A_{in,s} \right)^{\frac{\theta_s}{1+\theta_s}} \right]^{\frac{1+\theta_s}{\theta_s}}. \tag{K.4}$$

The unit expenditure function for this second composite is

$$\tilde{P}_{n,s} = \left(\sum_i \tilde{P}_{in,s}^{-\theta_s} \right)^{-1/\theta_s}. \tag{K.5}$$

Substituting (K.3) into (K.5) recovers (K.1). Scaling the sectoral quantity composite $Q^A_{n,s}$ by $1/\tilde{\Gamma}_s$ multiplies its unit-expenditure function by $\tilde{\Gamma}_s$, giving the Ricardian sectoral price index $P_{n,s}$ in (4).

CES expenditure shares also match. Cost minimization within (K.2) assigns the location-l product the conditional share

$$\frac{[\tilde{\gamma}_{il,s} \tilde{\tau}_{ln,s} (1 + t_{ln,s})]^{-\frac{\theta_s}{1-\rho}}}{\sum_m [\tilde{\gamma}_{im,s} \tilde{\tau}_{mn,s} (1 + t_{mn,s})]^{-\frac{\theta_s}{1-\rho}}} = \frac{D_{iln,s}}{D_{in,s}}.$$

Cost minimization across technology-source composites in (K.4) assigns source i the share

$$\frac{\tilde{P}_{in,s}^{-\theta_s}}{\sum_j \tilde{P}_{jn,s}^{-\theta_s}} = \frac{D_{in,s}^{1-\rho}}{D_{n,s}}.$$

Multiplying these two shares yields $\lambda_{iln,s}$ in (7). The deterministic Armington system therefore reproduces the Ricardian sectoral price index and source–location expenditure shares.